THE DETERMINATION OF A SPECTROSCOPIC BINARY ORBIT*

JOHN B. IRWIN
Goethe Link Observatory, Indiana University
Received March 27, 1952

ABSTRACT

Tables are presented that enable the computer to obtain quickly from the velocity-curve an accurate solution for the orbital elements of a spectroscopic binary. Applications to the velocity-curves of Boss 4496 and ι Arietis are given.

The most frequently used method of orbit computation for a spectroscopic binary is probably that of Lehmann-Filhés.[1] Although this method is fairly rapid and is of considerable analytical elegance, the accuracy of the solution depends not only on observations over the *entire* radial-velocity-curve but also on the accurate determination of the *times* of maximum and minimum velocity. The solution is weak, therefore, if these times are poorly fixed by the observations or if sections of the velocity-curve are poorly observed or not observed at all. This latter situation occurs quite frequently, for example, in the case of double-lined binaries at those times when the spectral lines of the two components cannot be separately measured. Furthermore, once the orbital elements have been determined by this method and a theoretical curve drawn, it is not immediately obvious to an inexperienced computer—if the fit with the observations is not completely satisfactory—as to just what adjustments to the elements should be made. In addition, it is not possible to estimate accurately the uncertainties or probable errors of the derived elements.

The method to be presented here is based on techniques somewhat similar to those to be found in the well-known Russell method for the solution of eclipsing binary light-curves. Attention will be focused on certain points or "levels" of the velocity-curve, and tables have been computed and are here presented which enable the computer to obtain very quickly a solution which need be based upon only the best-observed portions of the velocity-curve.

The observed velocity, V_o, is given by the well-known equation:[2,3]

$$V_o = \gamma + \frac{dz}{dt} = \gamma + K\,[\cos(v+\omega) + e\cos\omega]\,, \tag{1}$$

where

$$K = \frac{a\sin i}{13{,}750\,P\,(1-e^2)^{1/2}}\,; \tag{2}$$

γ is the velocity of the system, K is in km/sec, $a \sin i$ is in kilometers, and P is in mean solar days.

The mean velocity, V_m, is defined by

$$V_m = \tfrac{1}{2}\,(V_{\max} + V_{\min}) = \gamma + K\,e\cos\omega\,. \tag{3}$$

Eliminating γ between equations (1) and (3), we have:

$$\cos(v+\omega) = \frac{V_o - V_m}{K}\,. \tag{4}$$

* *Publications of the Goethe Link Observatory, Indiana University*, No. 7.

[1] *A.N.*, **136**, 17, 1894.

[2] J. A. Hynek, *Astrophysics* (New York: McGraw-Hill Book Co., 1951), chap. x.

[3] R. G. Aitken, *The Binary Stars* (New York: McGraw-Hill Book Co., 1935), chap. vi.

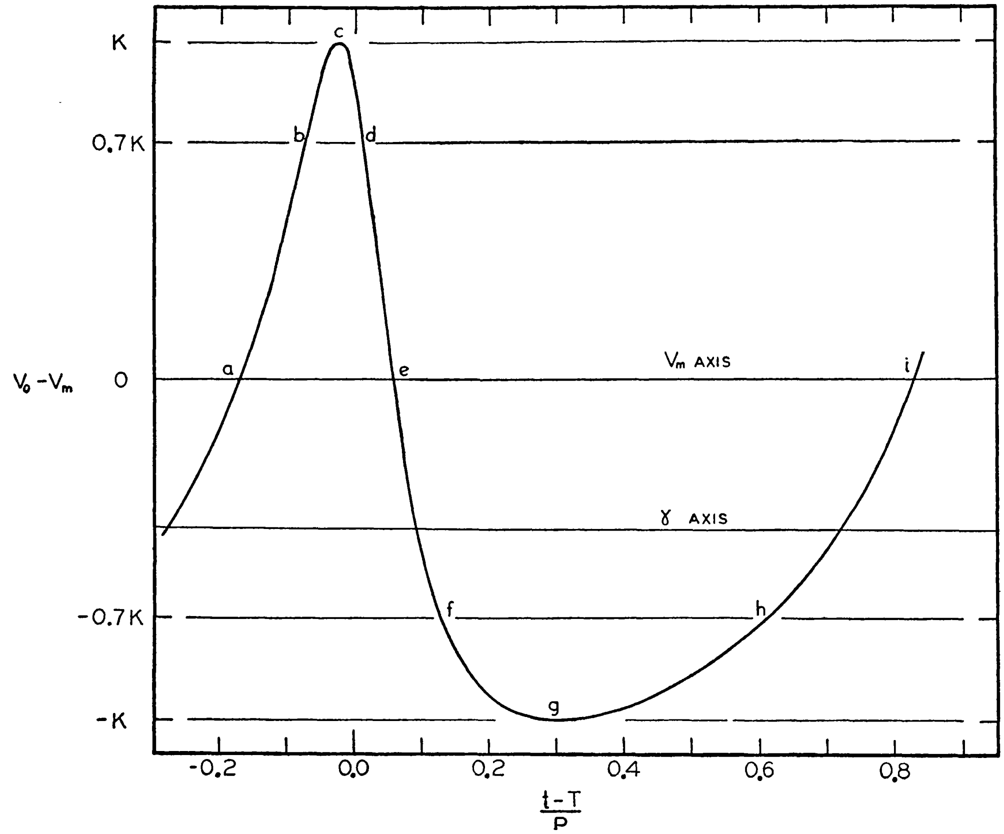

FIG. 1.—The times corresponding to the points a to i are used to calculate the nine parameters

TABLE 1

$$h = \frac{t_e - t_a}{P}$$

ω	0	15	30	45	60	75	90	105	120	135	150	165	180
ω	360	345	330	315	300	285	270	255	240	225	210	195	180
e													
0.00. .	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
.10. .	.436	.439	.445	.455	.468	.483	.500	.517	.532	.545	.555	.561	.564
.20. .	.374	.378	.390	.409	.435	.466	.500	.534	.565	.591	.610	.622	.626
.30. .	.312	.318	.335	.363	.402	.448	.500	.552	.598	.637	.665	.682	.688
.40. .	.252	.259	.280	.315	.365	.429	.500	.571	.635	.685	.720	.741	.748
.50. .	.196	.203	.225	.265	.326	.406	.500	.594	.674	.735	.775	.797	.804
.60. .	.142	.149	.171	.213	.281	.379	.500	.621	.719	.787	.829	.851	.858
.65. .	.118	.124	.145	.186	.256	.363	.500	.637	.744	.814	.855	.876	.882
.70. .	.094	.100	.119	.158	.229	.345	.500	.655	.771	.842	.881	.900	.906
.75. .	.072	.077	.094	.130	.199	323	500	677	801	.870	.906	.923	.928
.80. .	.052	.056	.070	.100	.166	.296	.500	.704	.834	.900	.930	.944	.948
.85. .	.034	.037	.047	.071	.128	.261	.500	.739	.872	.929	.953	.963	.966
.90. .	.0187	.0204	.027	.043	.086	.211	.500	.789	.914	.957	.973	.980	.981
0.95. .	0.0067	0.0073	0.0099	0.0168	0.039	0.133	0.500	0.867	0.961	0.983	0.990	0.993	0.993

Cos $(v + \omega)$ varies between ± 1, and the times corresponding to any predetermined value of the quantity $(V_o - V_m)/K$ can be read off the *observed* velocity-curve. For such predetermined values the angle $v + \omega$ can be *calculated* from equation (4), and, furthermore, for any assumed e and ω the values of $(t - T)/P$ can be calculated by means of the well-known relations[4]

$$\tan \tfrac{1}{2}E = \left(\frac{1-e}{1+e}\right)^{1/2} \tan \tfrac{1}{2} v \tag{5}$$

and

$$\frac{t-T}{P} = \frac{M}{2\pi} = \frac{E - e \sin E}{2\pi} . \tag{6}$$

The predetermined values of $(V_o - V_m)/K$ are here chosen to be 0.0, ± 0.7, and ± 1.0. These points on the velocity-curve are approximately equally spaced in time if the eccentricity is small. The following dimensionless parameters that have been chosen are functions of e and ω only:

$$h = \frac{t_e - t_a}{P} ; \tag{7}$$

$$s_1 = \frac{t_b - t_a}{P}, \quad s_2 = \frac{t_e - t_d}{P}, \quad s_3 = \frac{t_f - t_e}{P}, \quad s_4 = \frac{t_i - t_h}{P} ; \tag{8}$$

$$p_1 = \frac{t_c - t_b}{P}, \quad p_2 = \frac{t_d - t_b}{P}, \quad p_3 = \frac{t_g - t_f}{P}, \quad p_4 = \frac{t_h - t_f}{P} . \tag{9}$$

Here t_a, t_e, and t_i refer to points on the V_m axis; t_b and t_d refer to points where $(V_o - V_m)/K = 0.7$; t_f and t_h refer to points where $(V_o - V_m)/K = -0.7$; and t_c and t_g refer to the times of maximum and minimum velocities, respectively. These points are graphically illustrated in Figure 1.

The nine parameters defined in equations (7), (8), and (9) are tabulated in Tables 1-4 as functions of e and ω. The tables were derived from equation (4) by first calculating v, the true anomaly, for the predetermined values of $(V_o - V_m)/K$ and for values of ω every 15° from 0° to 90°, inclusive. The mean anomalies were next calculated (in degrees), and the final tables formed by subtracting appropriate mean anomalies and dividing by 360. The calculations were not particularly laborious. The accuracy of these tables is better than 0.0006 when three decimal places are given and 0.00006 when four places are given. It has been found possible, because of symmetry, to condense the tabulation of these nine parameters into only four tables. Table 3 may not be useful at all times because of the occasional difficulty in accurately locating the points c and g. This table may be of great usefulness in special cases, such as σ Piscium,[5] where only a small fraction of the velocity-curve is observable. A combination of any two of the nine parameters should usually suffice to determine e and ω.

INSTRUCTIONS TO THE COMPUTER

Given the observations and the velocity-curve drawn with the value of P assumed as known, the first steps are to fix the V_m axis as defined by equation (3) and to determine K from:

$$K = (V_{\max} - V_{\min}) . \tag{10}$$

The points a, b, c, d, e, f, g, h, and i are located as indicated in Figure 1, the times for these points read off the velocity-curve where possible, and the nine parameters calcu-

[4] For $e \leqq 0.77$ the Allegheny tables (*Pub. Allegheny Obs.*, Vol. 2, No. 17, 1912) will be found very convenient to use, rather than making use of eqs. (5) and (6).

[5] E. P. Belserene, *Ap. J.*, **105**, 229, 1947; *Lick Obs. Contr.*, Ser. II, No. 16.

lated as defined by equations (7), (8), and (9). As previously indicated, it may not be possible in the case of a double-lined binary to fix any or all of h, s_1, s_2, s_3, or s_4; or it may not be possible to fix points c and g—and hence p_1 and p_3—with any degree of accuracy from the observations.[6] For each known parameter a curve is plotted as shown in Figure 2, where e is the ordinate and ω the abscissa. The curve for each parameter is most easily

TABLE 2

$$s_1=\frac{t_b-t_a}{P};\qquad s_2=\frac{t_e-t_d}{P};\qquad s_3=\frac{t_f-t_e}{P};\qquad s_4=\frac{t_i-t_h}{P}$$

ω for s_1	0	15	30	45	60	75	90	105	120	135	150	165	180
ω for s_2	360	345	330	315	300	285	270	255	240	225	210	195	180
ω for s_3	180	195	210	225	240	255	270	285	300	315	330	345	360
ω for s_4	180	165	150	135	120	105	90	75	60	45	30	15	0
e													
0.00..	0.123	0.123	0.123	0.123	0.123	0.123	0.123	0.123	0.123	0.123	0.123	0.123	0.123
.10..	.113	.119	.125	.131	.138	.143	.147	.149	.149	.147	.143	.137	.131
.20..	.101	.111	.123	.137	.150	.163	.173	.178	.178	.173	.163	.150	.136
.30..	.088	.101	.118	.138	.161	.184	.202	.213	.213	.202	.183	.160	.137
.40..	.073	.088	.109	.136	.168	.204	.235	.253	.253	.234	.202	.167	.134
.50..	.058	.074	.096	.128	.171	.222	.271	.302	.301	.269	.220	.169	.126
.60..	.044	.057	.079	.114	.166	.237	.312	.362	.360	.309	.234	.164	.112
.65..	.037	.049	.070	.105	.161	.242	.334	.397	.396	.330	.238	.158	.103
.70..	.030	.041	.060	.093	.152	.244	.357	.438	.436	.353	.240	.149	.092
.75..	.023	.032	.049	.080	.139	.243	.381	.486	.484	.376	.238	.136	.078
.80..	.0167	.024	.038	.065	.122	.236	.407	.543	.540	.400	.230	.119	.063
.85..	.0110	.0162	.026	.048	.099	.220	.433	.613	.609	.424	.213	.096	.047
.90..	.0061	.0092	.0154	.030	.070	.188	.459	.703	.698	.447	.181	.067	.029
0.95..	0.0022	0.0034	0.0059	0.0123	0.033	0.124	0.484	0.827	0.822	0.465	0.117	0.032	0.0119

ω for s_1	180	195	210	225	240	255	270	285	300	315	330	345	360
ω for s_2	180	165	150	135	120	105	90	75	60	45	30	15	0
ω for s_3	0	15	30	45	60	75	90	105	120	135	150	165	180
ω for s_4	360	345	330	315	300	285	270	255	240	225	210	195	180
e													
0.00..	0.123	0.123	0.123	0.123	0.123	0.123	0.123	0.123	0.123	0.123	0.123	0.123	0.123
.10..	.131	.125	.119	.113	.108	.105	.102	.101	.101	.102	.105	.108	.113
.20..	.136	.123	.111	.101	.093	.087	.083	.082	.082	.083	.087	.093	.101
.30..	.137	.117	.100	.087	.078	.071	.066	.064	.064	.067	.071	.078	.088
.40..	.134	.108	.088	.073	.062	.056	.051	.049	.049	.052	.056	.063	.073
.50..	.126	.095	.073	.058	.048	.042	.038	.036	.036	.038	.042	.048	.058
.60..	.112	.078	.057	.043	.035	.030	.027	.025	.025	.027	.030	.035	.044
.65..	.103	.069	.048	.036	.029	.024	.022	.0204	.0204	.022	.024	.029	.037
.70..	.092	.059	.040	.029	.023	.0191	.0170	.0160	.0160	.0170	.0192	.023	.030
.75..	.078	.048	.032	.023	.0175	.0144	.0128	.0120	.0120	.0128	.0145	.0176	.023
.80..	.063	.037	.024	.0165	.0126	.0103	.0090	.0085	.0085	.0091	.0103	.0127	.0167
.85..	.047	.026	.0159	.0109	.0082	.0066	.0058	.0054	.0054	.0058	.0067	.0083	.0110
.90..	.029	.0151	.0090	.0060	.0045	.0036	.0031	.0029	.0029	.0031	.0036	.0045	.0061
0.95..	0.0119	0.0058	0.0033	0.0022	0.0016	0.0013	0.0011	0.0010	0.0010	0.0011	0.0013	0.0016	0.0022

[6] The determination of t_c and t_g may also be time-consuming and, for that reason, may often be omitted with little loss of accuracy. The technique of determination would be similar to the fixing of the date of maximum or minimum light of a variable star. See, for example, Campbell and Jacchia, *The Story of Variable Stars* (Philadelphia: Blakiston Co., 1941), Fig. 22.

calculated by entering the appropriate table and either interpolating for e at the various tabular values of ω or vice versa. The curves so plotted, because of observational errors, will not all meet at a point. The computer must here use his own judgment—with due regard to the nature of his observational material—as to what final values of e and ω to adopt. It may be convenient to take the curves in groups of two or three or possibly use the mean of only those intersections that are strongly determinate.

TABLE 3

$$p_1 = \frac{t_c - t_b}{P}; \qquad p_3 = \frac{t_g - t_f}{P}$$

ω for p_1 ω for p_3	0 180	15 195	30 210	45 225	60 240	75 255	90 270	105 285	120 300	135 315	150 330	165 345	180 360
e													
0.00..	0.127	0.127	0.127	0.127	0.127	0.127	0.127	0.127	0.127	0.127	0.127	0.127	0.127
.10..	.105	.108	.111	.116	.122	.128	.135	.141	.147	.151	.153	.153	.151
.20..	.086	.090	.096	.104	.114	.126	.140	.154	.167	.177	.183	.183	.177
.30..	.068	.073	.080	.090	.104	.121	.142	.165	.188	.207	.218	.218	.207
.40..	.053	.057	.065	.075	.091	.112	.139	.173	.209	.241	.259	.259	.239
.50..	.039	.043	.050	.060	.076	.098	.131	.175	.228	.278	.309	.308	.276
.60..	.027	.031	.036	.045	.059	.082	.117	.171	.243	.319	.370	.369	.316
.65..	.022	.025	.030	.038	.050	.072	.108	.165	.248	.342	.406	.405	.338
.70..	.0175	.0197	.024	.030	.042	.061	.096	.156	.251	.366	.448	.446	.361
.75..	.0132	.0149	.0181	.024	.033	.050	.083	.144	.250	.391	.496	.493	.385
.80..	.0093	.0106	.0130	.0172	.025	.039	.067	.126	.244	.417	.553	.550	.410
.85..	.0060	.0069	.0085	.0114	.0167	.027	.050	.103	.228	.445	.623	.620	.436
.90..	.0032	.0037	.0046	.0063	.0095	.0160	.031	.073	.196	.474	.713	.708	.461
0.95..	0.0011	0.0013	0.0016	0.0023	0.0035	0.0061	0.0128	0.035	0.132	0.504	0.836	0.831	0.485

ω for p_1 ω for p_3	180 0	195 15	210 30	225 45	240 60	255 75	270 90	285 105	300 120	315 135	330 150	345 165	360 180
e													
0.00..	0.127	0.127	0.127	0.127	0.127	0.127	0.127	0.127	0.127	0.127	0.127	0.127	0.127
.10..	.151	.146	.141	.135	.128	.122	.116	.111	.107	.105	.104	.104	.105
.20..	.177	.167	.154	.140	.126	.114	.103	.095	.089	.086	.084	.084	.086
.30..	.207	.187	.164	.141	.120	.103	.090	.080	.073	.068	.066	.066	.068
.40..	.239	.207	.171	.138	.111	.090	.075	.064	.057	.053	.051	.051	.053
.50..	.276	.226	.173	.130	.097	.075	.060	.049	.043	.039	.037	.037	.039
.60..	.316	.240	.169	.116	.081	.058	.045	.036	.030	.027	.026	.026	.027
.65..	.338	.245	.163	.106	.071	.050	.037	.029	.025	.022	.021	.021	.022
.70..	.361	.247	.154	.094	.060	.041	.030	.024	.0196	.0174	.0164	.0164	.0175
.75..	.385	.245	.141	.081	.049	033	023	0180	.0148	.0131	.0123	.0123	.0132
.80..	.410	.238	.123	.066	.038	.024	.0170	.0129	.0106	.0093	.0087	.0087	.0093
.85..	.436	.221	.100	.049	.027	.0164	.0112	.0084	.0068	.0060	.0056	.0056	.0060
.90..	.461	.189	.070	.030	.0156	.0093	.0062	.0046	.0037	.0032	.0030	.0030	.0032
0.95..	0.485	0.125	0.033	0.0124	0.0060	0.0034	0.0022	0.0016	0.0013	0.0011	0.0010	0.0010	0.0011

Using the adopted values of e and ω, the computer may now find it advantageous to extract from the tables new values of all nine parameters. Using these values and any one of the more strongly determined points on the velocity-curve, computed values of the nine points a to i can be plotted and compared with the observations. If the agreement is not entirely satisfactory, the tables will indicate what changes in e and ω are necessary and possible. A horizontal shift (or correction) in the time scale may conveniently be made here. A correction to either or both of the maximum or minimum velocities, and hence K and V_m, can also be quickly made.

The velocity of the system, γ, is now obtained from

$$\gamma = V_m - K e \cos \omega , \tag{11}$$

and the time of periastron passage, T, can be obtained from

$$V_p = V_m + K \cos \omega , \tag{12}$$

where V_p is the radial velocity at that time, that is, when $v = 0°$. Two points of the velocity-curve will have this ordinate; but, because $v + \omega = 0°$ at point c and $v + \omega =$

TABLE 4

$$p_2 = \frac{t_d - t_b}{P} ; \qquad p_4 = \frac{t_h - t_f}{P}$$

ω for p_2	0	15	30	45	60	75	90	105	120	135	150	165	180
ω for p_4	180	165	150	135	120	105	90	75	60	45	30	15	0
e													
0.00..	0.253	0.253	0.253	0.253	0.253	0.253	0.253	0.253	0.253	0.253	0.253	0.253	0.253
.10..	.210	.211	.215	.221	.229	.239	.251	.263	.275	.285	.294	.299	.301
.20..	.171	.173	.179	.189	.204	.222	.244	.268	.293	.317	.337	.350	.354
.30..	.137	.139	.146	.158	.176	.200	.231	.268	.308	.348	.382	.405	.413
.40..	.106	.108	.115	.128	.148	.176	.214	.263	.319	.378	.431	.466	.479
.50..	.079	.081	.087	.099	.119	.148	.191	.250	.325	.407	.482	.534	.552
.60..	.055	.057	.062	.072	.090	.117	.162	.229	.324	.435	.538	.609	.633
.65..	.044	.046	.051	.060	.075	.101	.145	.215	.319	.448	.569	.649	.677
.70..	.035	.036	.040	.048	.061	.085	.126	.198	.312	.460	.601	.693	.723
.75..	.026	.027	.030	.037	.048	.068	.106	.176	.300	.472	.637	.739	.771
.80..	.0186	.0194	.022	.026	.035	.052	.084	.151	.282	.483	.676	.788	.821
.85..	.0120	.0125	.0141	.0173	.024	.036	.061	.120	.255	.494	.724	.841	.872
.90..	.0065	.0067	.0076	.0095	.0132	.0206	.037	.082	.212	.504	.783	.897	.923
0.95..	0.0023	0.0024	0.0027	0.0034	0.0048	0.0077	0.0151	0.038	0.138	0.516	0.869	0.955	0.969

ω for p_2	180	195	210	225	240	255	270	285	300	315	330	345	360
ω for p_4	360	345	330	315	300	285	270	255	240	225	210	195	180
e													
0.00..	0.253	0.253	0.253	0.253	0.253	0.253	0.253	0.253	0.253	0.253	0.253	0.253	0.253
.10..	.301	.299	.294	.285	.275	.263	.251	.239	.229	.221	.215	.211	.210
.20..	.354	.350	.337	.317	.293	.268	.244	.222	.204	.189	.179	.173	.171
.30..	.413	.405	.382	.348	.308	.268	.231	.200	.176	.158	.146	.139	.137
.40..	.479	.466	.431	.378	.319	.263	.214	.176	.148	.128	.115	.108	.106
.50..	.552	.534	.482	.407	.325	.250	.191	.148	.119	.099	.087	.081	.079
.60..	.633	.609	.538	.435	.324	.229	.162	.117	.090	.072	.062	.057	.055
.65..	.677	.649	.569	.448	.319	.215	.145	.101	.075	.060	.051	.046	.044
.70..	.723	.693	.601	.460	.312	.198	.126	.085	.061	.048	.040	.036	.035
.75..	.771	.739	.637	.472	.300	.176	.106	.068	.048	.037	.030	.027	.026
.80..	.821	.788	.676	.483	.282	.151	.084	.052	.035	.026	.022	.0194	.0186
.85..	.872	.841	.724	.494	.255	.120	.061	.036	.024	.0173	.0141	.0125	.0120
.90..	.923	.897	.783	.504	.212	.082	.037	.0206	.0132	.0095	.0076	.0067	.0065
0.95..	0.969	0.955	0.869	0.516	0.138	0.038	0.0151	0.0077	0.0048	0.0034	0.0027	0.0024	0.0023

180° at point g, there should be no ambiguity unless T occurs at, or very close to, the time of V_{max} or V_{min}. For such cases it may be advisable to determine the value of v at the point a, where $v + \omega = 270°$, or at the point e,[7] where $v + \omega = 90°$, and find T by determining M for this v, either using equations (5) and (6) or the Allegheny tables.[4] A final representation may now be made with the aid of equation (1). If the computer at this point feels that the observational material is sufficiently good, he should proceed to a least-squares differential correction.

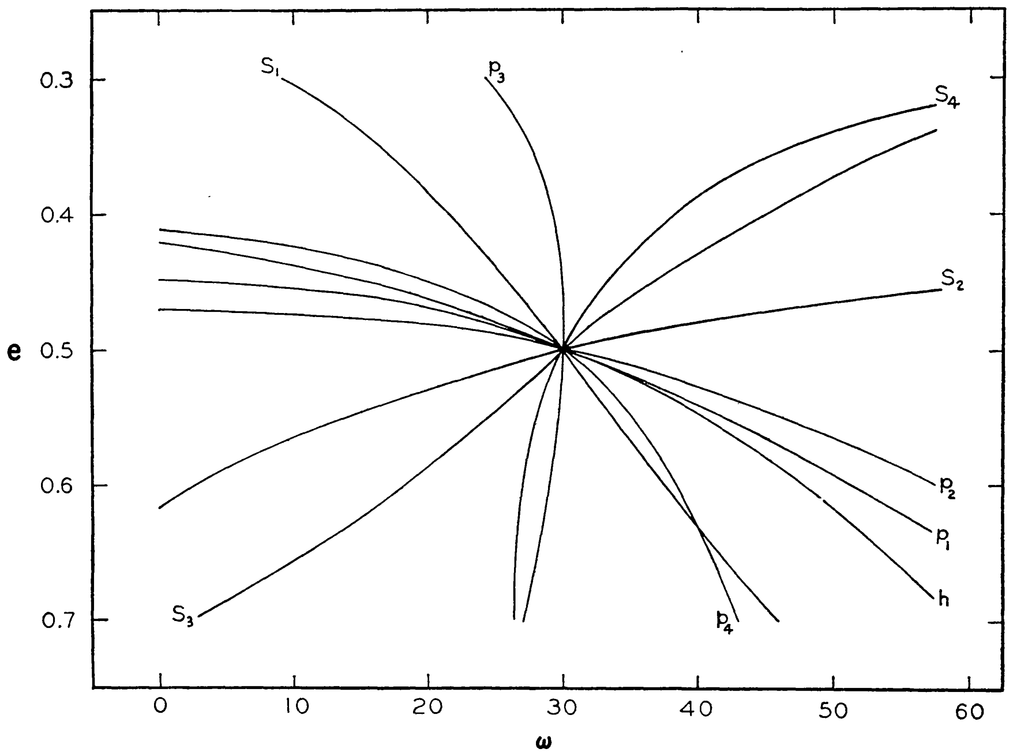

FIG. 2.—The "theoretical" solution of the velocity-curve shown in Figure 1

The above procedure may be modified slightly in the case of a double-lined spectroscopic binary. A graphical or least-squares solution for γ and the mass ratio might first be made by the ingenious method proposed by O. C. Wilson.[8] Knowing γ, a determination of $e \cos \omega$ could be made from

$$e \cos \omega = \frac{V_m - \gamma}{K}. \tag{13}$$

[7] Or at any other of those points where $\cos (v + \omega)$ is accurately known.

[8] *Ap. J.*, **93**, 29, 1941. In Wilson's notation the mass ratio is $r = m_u/m_v = -dv/du$ and r is determined from the negative slope of the straight line defined by the observations plotted on the u–v plane, where u and v are the observed velocities of the respective components. Strictly speaking, the form of Wilson's least-squares equation of condition: $\gamma(1 + r) - ur = v$, is not entirely correct, inasmuch as there are observational errors in *both* co-ordinates. The analysis may be further complicated by the fact that the u velocities may have different weights than the v velocities. Perhaps a better result might be obtained if two solutions were to be made, the second one using the equation of condition: $\gamma(1 + r)/r - v/r = u$. An appropriately weighted mean of the results of the two solutions could then be taken. A more refined—but lengthier—technique, if the u and v velocities are of equal weight, would be to minimize the sum of the squares of the residuals perpendicular to the line. I am indebted to Mr. Arthur N. Cox for bringing these points to my attention.

This value of $e \cos \omega$ could be expressed as a curve on the ω–e plot and would assist in the solution for these two quantities. The rest of the solution would proceed as before, either after appropriately "reflecting" one set of velocities about the γ axis or by making separate solutions for the two velocity-curves and combining the results with appropriate weights, remembering that ω for one star is 180° different from the ω for the other star.

An illustration of the above method is shown in Figure 3, where a solution is made for e and ω for the double-lined spectroscopic binary Boss 4496.[9] A preliminary determination of γ and r was made by the method of O. C. Wilson[8] and $e \cos \omega$ determined from equation (13) and plotted on the figure. Average values of the various parameters were

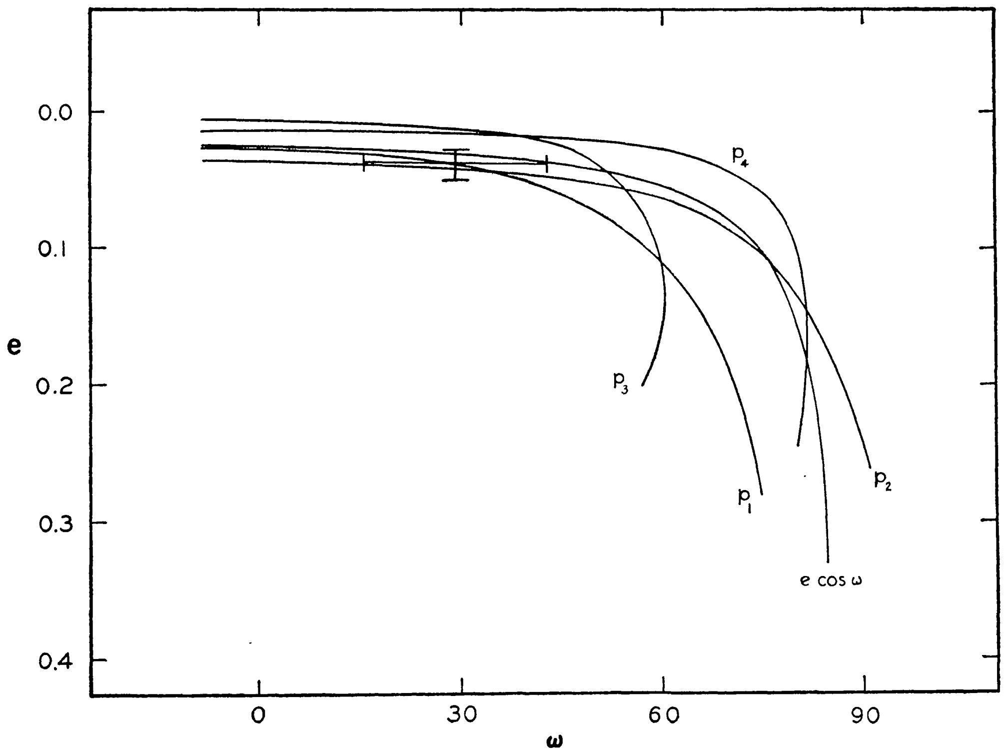

FIG. 3.—The solution for Boss 4496. The least-squares solution gives $e = 0.04 \pm 0.011$, $\omega = 29° \pm 13{}^{\circ}\!.7$. The indeterminacy of ω is clearly shown. The positions of the curves vary markedly with small changes in the values of the parameters.

used as derived from the *two* velocity-curves. The least-squares values of e and ω are shown with their respective probable errors. The indeterminacy of ω is not primarily due to the method or to the observations but, as is well known,[10] is a fundamental feature of nearly circular orbits. In general, the *amount* of distortion (from a sine curve) is primarily dependent on the value of e, whereas how, or at what point, that distortion appears in the velocity-curve depends on ω. If the distortion is at or below the limit of detection, then ω—and also T—must necessarily be quite indeterminate. The technique of analysis in such cases would seem to be to assume $e = 0$ and to proceed with a least-squares solution by the method of Sterne,[11] where e, ω, and T are essentially replaced by $e \sin \omega$,

[9] J. Sahade and J. L. Dessy, *Ap. J.*, **115**, 53, 1952.

[10] W. J. Luyten, *Pub. Obs. U. Minnesota*, **2**, 53, 1936; *Ap. J.*, **84**, 85, 1936.

[11] *Harvard Repr.*, No. 222; *Proc. Nat. Acad. Sci.*, **27**, 175, 1941.

$e \cos \omega$, and T_0, where T_0 represents the time at which the mean longitude $M + \omega$ is equal to zero. Figure 4 illustrates the solution of the velocity-curve of ι Arietis,[12] a single-lined spectroscopic binary of moderate eccentricity. The least-squares values of e and ω, together with their probable errors, are shown in the figure.

The advantages of the precomputed method presented in this paper would seem to be (1) its accuracy, (2) its rapidity, (3) the possibility of making adjustments in the elements *before* the final representation, (4) the fact that the uncertainties in the adopted values of e and ω are indicated on the ω–e plot, and (5) the method's ability to make use

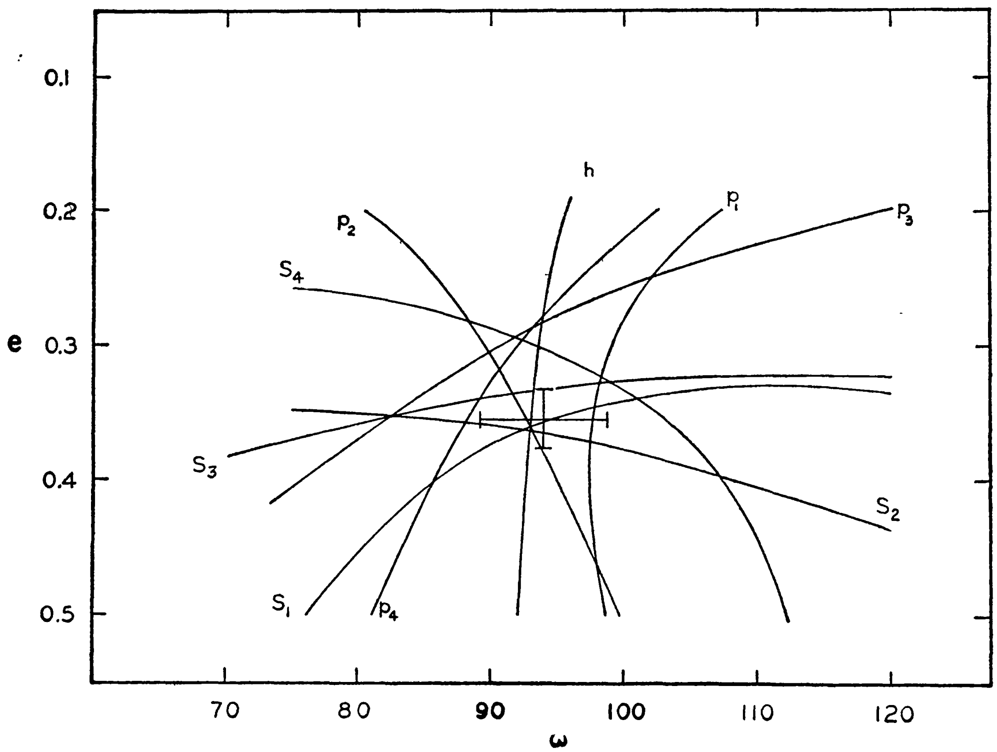

FIG. 4.—The solution for ι Arietis. The least-squares solution gives: $e = 0.356 \pm 0.022$, $\omega = 94\overset{\circ}{.}04 \pm 4\overset{\circ}{.}72$.

of only the well-observed portions of the velocity-curve. For these reasons this method would seem to have very definite advantages in comparison to the method of Lehmann-Filhés. At those observatories, however, where a standard set of velocity-curves[13] is available and is used in routine fashion for the determination of preliminary elements preparatory to a least-squares correction, the method presented here would seem to be somewhat slower and more complex.

I would like to express my thanks to Drs. W. J. Luyten, R. M. Petrie, and O. Struve for a number of valuable comments and suggestions.

[12] K. C. Gordon, *Ap. J.*, **103**, 16, 1946; *Lick Obs. Contr.*, Ser. II, No. 12.

[13] E. S. King, *Harvard Ann.*, **81**, 231, 1920.