MODELS OF WOLF-RAYET STARS

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ABSTRACT The current status of our knowledge about formation, structure and evolution of Wolf-Rayet stars is reviewd, with emphasis on a discussion of corresponding stellar models. The relevance of the LBV-scenario for WR star formation is outlined. The effect of the presence of hydrogen in a WR envelope is discussed, together with observational consequences. Hydrogenless WR stars are shown to closely follow simple relations for the dependence of luminosity, radius, and surface temperature as a function of their mass. The use of these relations for simplified WR evolution calculations is demonstrated. The concept of mass dependent mass loss rates for hydrogenless WR stars is motivated and its consequences are discussed. Surface abundance predictions for the different WR types are discussed, with special emphasis to the "WN+WC" spectral type. Along with our discussion we present some details concerning the WR phase of a recent $60\,M_\odot$ evolutionary calculation, which was computed with the same input physics which reproduced the progenitor evolution of SN 1987A in a $20\,M_\odot$ case, and which may be a representative case concerning WR stars in many respects.

INTRODUCTION

Wolf-Rayet (WR) stars have many extreme properties: they belong to the most luminous stars with luminosities typically between 10^5 and $10^6\,L_\odot$, and are extremely hot, which places them in the upper left corner of the HR diagram. Their intense stellar wind, which induces mass loss of several $10^{-5}\,M_\odot\,yr^{-1}$, is responsible for the dominance of emission lines in their spectra. Their surface composition is highly anomalous, dominated by helium rather than hydrogen, the latter being absent in many cases.

Because of the dense stellar wind, the hydrostatic surface of WR stars is mostly not accessible to direct observations, which makes it very difficult to attach stellar models to certain observed WR stars on the basis of their radii, surface temperatures or luminosities. Only spectroscopic determinations of abundances or abundance ratios of H, He, C, N, and O (Willis, 1982, Torres, 1988; Simth and Hummer, 1988) allowed to identify certain classes of theoretical stellar models as models for WR stars, namely those which contain a surface abundance pattern according to incomplete or complete hydrogen burning (WN stars) or incomplete helium burning (WC stars; cf. Maeder, 1983). Though both, the WN and WC class, are further divided into subtypes (cf. van der Hucht et al., 1981), this division can not be transmitted to stellar models in most cases. The only obvious trend concerns the WN stars, where hydrogen seems to be present to a substantial amount in so called "late types" (WNL), but absent in most "early types" WNs (WNE) (cf. Willis, 1982). Though this scheme seems to have exceptions (Hamann, this volume), we shall designate hydrogenless WN models as WNE stars and hydrogen containing ones as WNL stars in the following.

The WR-progenitor evolution and the WR formation process are presently not well understood. The identification of WR stars as objects which expose high quantities of new synthesized material at their surface allows basically two kinds of WR formation scenarios. In the first one, the ashes of nuclear burning is uncovered by heavy mass loss in the pre-WR stage, meaning that the surface of a WR stars was contained in the convective core of the progenitor stars. Pre-WR stages of heavy mass loss could be the red supergiant (RSG) stage, the Luminous Blue Variable (LBV) phase, or the Roche-lobe overflow phase in a close binary system (cf. Maeder, 1982). The second possibility to get the nuclei, which are formed in the stellar core, out to the surface, is mixing. Maeder (1987) proposed, that mixing due to differential rotation in fast spinning massive stars could keep them chemically homogeneous throughout their evolution, which leads to WR stars even without any mass loss. A combination of mass loss and extra mixing may work as well for producing WR stars, as demonstrated e.g. by Prantzos et al. (1986), who assumed a large increase of the convective core size ("convective overshooting") and adopted standard mass loss rates for their evolutionary computations. In the next section we shall discuss some theoretical models for the formation of WR stars.

Once WR stars are formed, their internal structure is essentially determined by the WR mass (Langer, 1989), and can therefore be discussed completely independent of problems involved in the pre-WR evolution. Since the WR mass is the dominant parameter, WR mass loss plays a prominent role during the evolution of WR stars, as described next in this paper. In the last section, we shall review surface abundances of WR models, with special emphasis to the recently discussed "WN+WC"-spectral type.

WR STAR FORMATION

In order to assess the most simple case, we shall confine the discussion in this section to "non-rotating, non-magnetic single stars"; it is not clear if this is also the most common case, but there is some hope. The rotational mixing model of Maeder (1987) is not likely to play a role for the majority of WR stars, since it would cause a significant extension of the main sequence band towards higher surface temperatures, which is not observed. The "blue stragglers", which can be explained in this scenario, are the exception, not the rule. Magnetic fields are not known to play a general role during the evolution of possible WR progenitors (O-stars, supergiants, LBVs). And binarity may certainly be involved in the formation of some WR stars, many of them, however, are supposed to be single stars (cf. Chiosi and Maeder, 1986). So, we are left with mass loss scenarios with or without some amount of "extra mixing".

Massive stars are known to have a stellar wind and mass loss already on the main sequence. Though the amount of mass lost during core hydrogen burning may be small compared to the pre-WR mass loss in later phases, its amount is relevant to the later evolution. In case the Schwarzschild criterion for convection is used, only relatively high mass loss rates will make the star evolve to cool surface temperatures after core hydrogen exhaustion (cf. e.g.

Stothers and Chin, 1979), which seems necessary in order to achieve a structure which allows a high mass loss rate (i.e. LBV, see below, or RSG). Computations performed with Lamers (1981) mass loss rate (cf. Langer and El Eid, 1986) showed, that this is high enough in order to obtain cool stars even for an initial mass as high as $100\,M_\odot$. However, in recent computations (Langer and El Eid, in prep.), relying for the first time on theoretical mass loss rates of Kudritzki et al. (1989), massive stars evolve towards cool temperatures only on a nuclear timescale.

There are two ways to solve this problem: either to perform more mixing as according to the Schwarzschild criterion (i.e. overshooting), or to perform less mixing, i.e. to take the effect of molecular weight gradients on convection into account (semiconvection). Both possibilities are known to lead to cool stellar surface temperatures after core hydrogen exhaustion on a thermal timescale (cf. Stothers and Chin, 1985; and Langer at al., 1985, for both processes, respectively). We tend to favor semiconvection from overshooting for the following reasons. It is theoretically motivated (Kato, 1966; Langer et al., 1983), and it explains in a natural way the blue-red-blue evolution of the SN 1987A progenitor together with the existence of both, blue and red supergiants in the LMC (Langer et al, 1989). It also leads to the observed WN+WC transition type stars, as discussed below. On the other side, the only observation, which seems to call for (a moderate amount of) overshooting for stars of the considered mass range is the large extension of the main sequence band to cool temperatures as observed in young open clusters (cf. Mermilliod and Maeder, 1986). However, both processes are not necessarily in contradiction, as long as the overshooting is restricted to regions in the star which are chemically homogeneous (i.e. basically to the core hydrogen burning phase), but models including both processes as described are yet to be computed.

Since no RSGs more luminous than $\sim 10^{5.7}\,L_\odot$ (corresponding roughly to $M_{ZAMS} = 50\,M_\odot$) are observed (Humphreys and McElroy, 1984), more luminous stars are supposed to loose their H-rich envelope during the LBV phase (cf. Maeder, 1983). Fig. 1 shows the evolutionary track of an initial $60\,M_\odot$ star of Population I composition in the HR diagram, which has been computed with mass loss according to Kudritzki et al. (1989) for the main sequence phase and with semiconvection according to Langer et al. (1983, 1985). For the LBV phase we adopted a mass loss rate as $\dot{M} = \dot{M}_0 \cdot (T_0/T_{eff})^x$, with $\dot{M}_0$ as a standard mass loss rate (we adopted the formula of de Jager et al., 1987), T_0 a temperature close to the Humphreys-Davidson limit of observed stars, and x a large number (we used $T_0 = 17\,000\,K$ and $x = 5$). Due to this formula, the mass loss rate increases drastically for effective temperatures below T_0. The stellar model stays for $\sim 10^4\,yr$ in the "forbidden region" of the HR diagram, reducing its mass to $27\,M_\odot$, thereby increasing the surface helium mass fraction to $Y_s = 0.75$, and then turns towards the WR region (see below). The same star but evolved with the Schwarzschild criterion for convection spends almost its entire He-burning lifetime at an effective temperature of $\sim 15\,000\,K$, which is incompatible with observations. (For discussion of observations and physics of LBVs, see: Davidson and Moffat, 1989).

We note the similarity of the HR diagram evolution of our model so far (Fig. 1) with that of the $60\,M_\odot$ sequence of Maeder and Meynet (1987), who included overshooting in their calculations.

Because of the existence of RSGs with luminosities below $\sim 10^{5.7}\,L_\odot$, stars

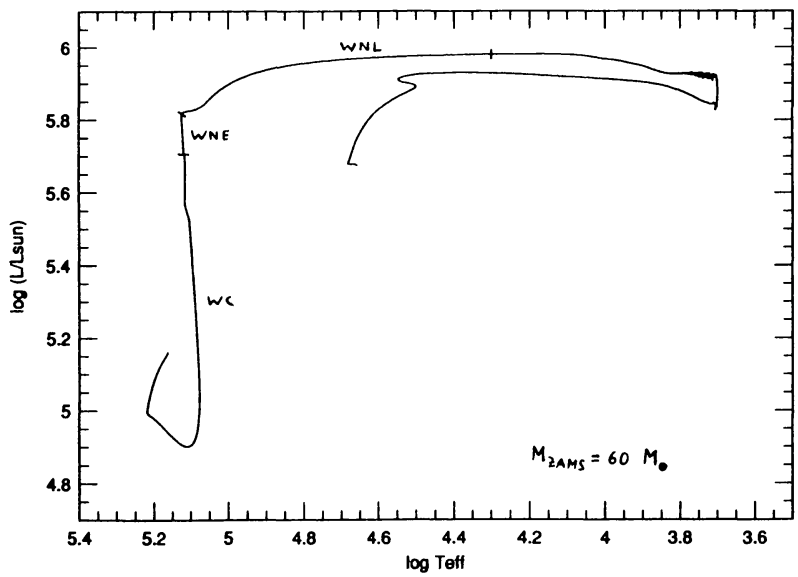

Figure 1. Evolutionary track of an initial $60\,M_\odot$ star of Population I composition in the HR diagram. Note that for the Wolf-Rayet phases the WR surface temperature rather than the temperature at optical depth 2/3 is displayed.

with ZAMS-masses below $\sim 50\,M_\odot$ are supposed to be able to evolve up to the Hayashi line, and the more massive ones are thought of as loosing sufficient mass there in order to evolve into WR stars (cf. Maeder and Meynet, 1987). Whether some of them evolve also through an LBV phase, and whether this would be before or after the RSG phase is unknown (see Langer, 1989b). The basic differences between WR progenitors which evolve through the RSG stage and those which do not is, that the latter ones turn into WR stars almost immediately at central helium ignition, since from the small number of observed LBVs we can conclude that the LBV phase cannot last very long (cf. Lamers, 1989). Stars which evolve to WRs through the RSG stage, however, may spend a large fraction of core helium burning at the RSG branch before turning into WR stars (cf. Fig. 2 of Langer, 1987). For this reason, stars with initial masses above $\sim 50\,M_\odot$ may statistically contribute more to the sample of observed WR stars than post-RSG WR stars, despite the rapid decline of the initial mass function (IMF) towards higher masses. This is as much more true for higher values of the lower ZAMS mass limit for WR formation.

We conclude that, in this respect, the LBV-scenario for WR formation may be the dominant one. Note that this is not necessarily the case concerning e.g. statistics of supernova progenitors, where the post-RSG WRs may have a similar importance as the post-LBV WRs (cf. Langer, 1990).

THE INTERNAL STRUCTURE OF WR STARS

In general, the structure of a WR star is determined primarily by its actual mass and can therefore be discussed independent of evolutionary considerations.

However, this is not quite true for the WNL stars, due to the presence of hydrogen, which affects the internal structure in several effects. Most important is the existence of a hydrogen burning shell in WNL models, which accounts for a second source of nuclear energy besides the He-burning stellar core. Due to this, and also because hydrogen matter has a high radiative opacity and a low mean molecular weight, the radii of WNL stars are considerably larger and their surface temperatures correspondingly smaller as compared to WNE and WC stars. This can be clearly seen in Fig. 1 and Fig 2a, where the different WR stages are indicated.

Since the optical thickness of the WR wind for a given mass loss rate is in first approximation proportional to the increase of the radius of the hydrostatic WR surface (cf. Langer, 1989), absorption lines are most likely to be found in spectra of the most hydrogenrich WN stars (or, of course, for WR stars with relatively low mass loss rates).

The temperature and density structure above the H-burning shell of a WNL star — and thereby also its radius — depend on the hydrogen profile (and thereby on the previous evolution) in two ways. The hydrogen concentration close to the burning shell influences the rate of nuclear energy generation, and the hydrogen mass fraction X_H determines the radiative opacity κ according to $\kappa \sim (1+X_H)$, since the opacity is mainly due to electron scattering. Figs. 1 and 2a demonstrate how the surface temperature of our $60\,M_\odot$ sequence increases from $\sim 20\,000\,K$ well above $100\,000\,K$ as the mass of the hydrogen envelope as well as the maximum (surface) hydrogen mass fraction are reduced from their initial values ($\sim 4\,M_\odot$ and $X_s = 0.23$) towards zero due to continuous mass loss, thereby approaching the WNE phase.

A further aspect of the presence of hydrogen is, that it can prevent the WR star from being vibrationally unstable according to the ϵ-mechanism (cf. Maeder, 1985). Since vibrational instability may lead to a high rate of mass loss (Appenzeller, 1970; Maeder, 1985), it is possible that WNL stars have a considerably smaller mass loss rate (due to radiation pressure?) compared to the vibrationally unstable WNE and WC stars of comparable mass.

The structure of WNE and WC stars is almost completely independent of the previous evolution. Though their internal composition profiles may be influenced to a certain extent by details of the progenitor evolution (e.g. the convective core size), it has almost no consequences, since those stars are so hot that their equation of state is radiation dominated, and the opacity is almost purely due to electron scattering, which is composition independent in the absence of hydrogen. Only in a tiny amount of mass close to the stellar surface, temperatures drop sufficiently (i.e. below $\sim 10^7\,K$) in order to allow a slight composition sensitivity (cf. Fig. 4 of Langer, 1989), which is responsible for the drop in surface temperature by $\sim 5\,000\,K$ at the transition from the WNE to the WC phase in our $60\,M_\odot$ sequence (cf. Fig. 2a), where the surface carbon abundance increases from 10^{-4} to $\sim$40% (see also Fig. 3, below).

The general insensitivity of the structure to the internal composition is the reason for the existence of a very narrow mass-luminosity relation for WNE and WC stars (cf. Maeder, 1983). Especially for WNE stars, which have a well defined surface composition for a given metallicity Z (e.g. $X_{He} \simeq 1$, $X_N \simeq 2/3Z$), the L-M relation should be extremely narrow, and also R-M and T-M relations are found (Langer, 1989)

In the next section it is shown, that the simple structure of hydrogenless

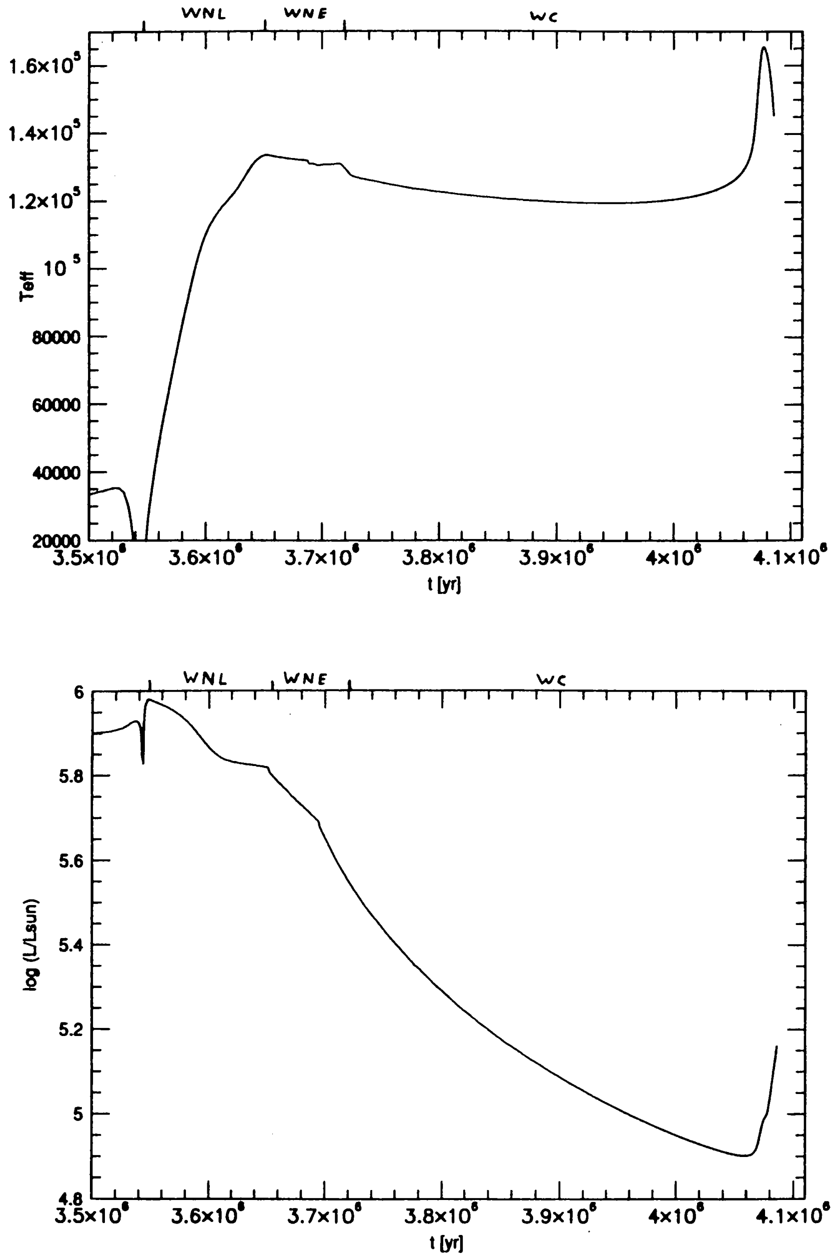

Figure 2. Time evolution of surface temperature (Fig. 2a) and luminosity (Fig 2b) for the post hydrogen burning evolution of our $60\,M_{\odot}$ sequence. See text for details.

WR stars allows to set up a very efficient way of computing their evolution.

EVOLUTION OF WR STARS

The first phase during the evolution of a newborn WR star is the WNL stage as a consequence of a shrinking convective core during central hydrogen burning. However, depending somewhat on the steepness of the hydrogen profile above the H-burning shell, the WNL stage may be rather short in WRs originating from the lower part of the considered ZAMS mass range, while its duration increases towards higher masses with the implication that the WNL phase may last for the whole post main sequence time for the highest ZAMS masses (Langer, 1987).

However, most WR stars are loosing completely their hydrogen envelope as indicated by the large number of observed WNE and WC stars (cf. van der Hucht et al., 1988). Once the hydrogen envelope is completely lost, the WR models strictly obey the mass-luminosity relation for hydrogenless WR stars, as outlined in the previous section. This allows — for a given mass loss rate for WR stars — the computations of all details of their further evolution by solving only the equation of global energy conservation (Langer, 1989a), since their mass determines their luminosity, which in turn determines the rate of nuclear burning in the stellar core. This simplification makes it possible to compute a very large number of WR evolution sequences and thus to perform reliable averages over the stellar IMF, which then allows a meaningful comparison to observed WR samples.

Using this method, Langer (1989a) has computed numerous sets of WR sequences, varying the basic remaining unknown quantity, i.e. the mass loss rate of WR stars. He showed that a constant WR mass loss rate, as usually adopted in stellar evolution calculations (cf. Maeder and Meynet, 1987), is incompatible with observed masses, luminosities, subtype number ratios and progenitor ZAMS masses, independent of the value of the constant mass loss rate. The dependence of the internal WR structure and of the WR surface conditions on their mass was the motivation for adopting mass dependent mass loss rates of the form $\dot{M}_{WR} \sim M_{WR}^{\alpha}$, α being a free parameter. Theoretical considerations (cf. Langer, 1989) as well as observed masses and mass loss rates (Abbott et al., 1986; cf. also Langer, 1989a) are compatible with $1 \lesssim \alpha \lesssim 4$ as the most probable range for α. Evolutionary computations showed, that IMF averaged quantities are often not very sensitive to this parameter as long as within the boundaries specified above, but are quite different from results obtained with $\alpha = 0$ (i.e. constant WR mass loss). Langer (1989a) concluded, that a mass loss rate of $\dot{M}_{WR} = f \cdot 10^{-7} (M_{WR}/M_{\odot})^{2.5}$ $[M_{\odot}\, yr^{-1}]$ for WNE and WC stars with $0.5 \lesssim f \lesssim 1$ yields a good agreement with observations.

Fig. 2b shows the time evolution of the luminosity of our $60\, M_{\odot}$ sequence during the WR stages, which is a consequence of the WR mass loss rate. We adopted the rate quoted above, with $f = 0.5$ for the WNE and $f = 1$ for the WC stage (cf. Langer, 1989a, for justification). Due to the mass-luminosity relation for hydrogenless WR stars, the rate of luminosity decrease reflects directly the rate of mass loss. Note that this is not the case for the WNL phase, for which we adopted a constant rate of mass loss of $5 \cdot 10^{-5}\, M_{\odot}\, yr^{-1}$. The main reason is a non-negligible contribution of the H-burning shell to the total luminosity in the early WNL phase. Mass loss rates of up to $10^{-4}\, M_{\odot}\, yr^{-1}$ are found for the WNE and early WC phase of our sequence, which is no contradiction to lower mass loss rates of the even more massive WNL progenitor as stated in the previous section. Such high values also fall well within the range of recent observational mass loss rate determinations for WNE and WC stars (de Freitas Pacheco and Machado, 1988; Schmutz et al, 1989).

As shown in Fig. 2a, and also evident from Fig. 1, the surface temperature stays almost constant during the WNE and WC phase, though the mass decreases from $22\,M_\odot$ to $6\,M_\odot$ in that time, in close agreement with the WR models of Langer (1989). Note that the temperature displayed in Fig. 2a as well as in Fig 1 is the temperature at the hydrostatic surface of the WR star, which may be much different from the temperature at an optical depth of 2/3 in the stellar wind (cf. discussion in Langer, 1989; especially his Fig. 2).

The final rapid variation of both, surface temperature and luminosity of our $60\,M_\odot$ sequence, which is visible in Fig. 2a and b, respectively, corresponds to the evolution beyond core helium exhaustion and is probably not relevant to observed WR stars because of its short duration. Its importance for a discussion of supernova events related with WR stars is discussed elsewhere (Langer, 1990).

SURFACE ABUNDANCES OF WR STARS

We have been identifying WR stellar models by their surface chemical composition, referring to models with partial and complete H-burning and partial He-burning surface composition as WNL, WNE and WC stars, respectively, according to the main trend emerging from spectroscopic abundance determinations (see above). Concerning the WR models, the relative distribution of the different isotopes within the ashes of a certain burning stage are mostly fixed by the nuclear physics. However, models for each subclass contain some theoretical uncertainty concerning surface abundances, except perhaps the WNE models.

Concerning WNL models, it is not generally known how large their surface hydrogen content may be. This is not so much a problem for each specific stellar model sequence, since the end of the LBV or RSG phase is usually well defined by a fast evolution of the star to high effective temperatures. However, there is no systematic study analyzing the dependence of the surface H-mass fraction at the beginning of the WNL phase from stellar and physical parameters. Our $60\,M_\odot$ sequence turns to the WNL stage at a surface H-mass fraction of 0.23, which is in good agreement with the $60\,M_\odot$ sequence of Maeder and Meynet (1987), who used quite different physics in their computations as discussed above. This gives some hope to the supposition that the dependence on the stellar physics is not large. Certainly, the limiting hydrogen mass fraction is sufficiently low in order to assure CN-equilibrium abundances as well as a substantial reduction of the oxygen abundance.

It is interesting to mention in this context, that significant hydrogen deficiency and nitrogen enrichment has been detected in some LBVs (see e.g. Davidson et al., 1984). A determination of the oxygen abundance in these cases would be most conclusive.

As for the WC stars, there are two interesting items relevant to the present context. The first concerns the abundance of He, C, and O at the beginning of the WC phase. Since the convective core is growing during early core He-burning before it is forced to shrink due to mass loss, high enhancements of carbon and oxygen may be present in the WC stage from the beginning. The He-burning convective core can only grow as long as an active hydrogen burning shell above shields it from the influence of mass loss, i.e. basically during the WNL phase. However, its growth may be stopped earlier due to large molecular weight barriers developing at its outer boundary. Therefore e.g. the C/He and O/He ratios at the beginning of the WC phase are very uncertain (see also discussion in Maeder and Meynet, 1987).

Unfortunately, these quantities (especially the O/C ratio) are affected by a large uncertainty concerning the nuclear physics, i.e. the $^{12}C(\alpha,\gamma)^{16}O$ reaction rate (cf. Caughlan and Fowler, 1988), which is still uncertain by a factor of ~3. For these reasons, no stringent constraints could be obtained from recent abundance determinations of WC stars (Torres, 1988; Smith and Hummer, 1988; de Freitas Pacheco and Machado, 1988), which also involve some uncertainty. However, we want to point out the extreme importance of such measurements; especially if helium, carbon, and —most important — oxygen abundances could be derived simultaneously for one object, it could put enormous constraints on both, stellar and nuclear physics.

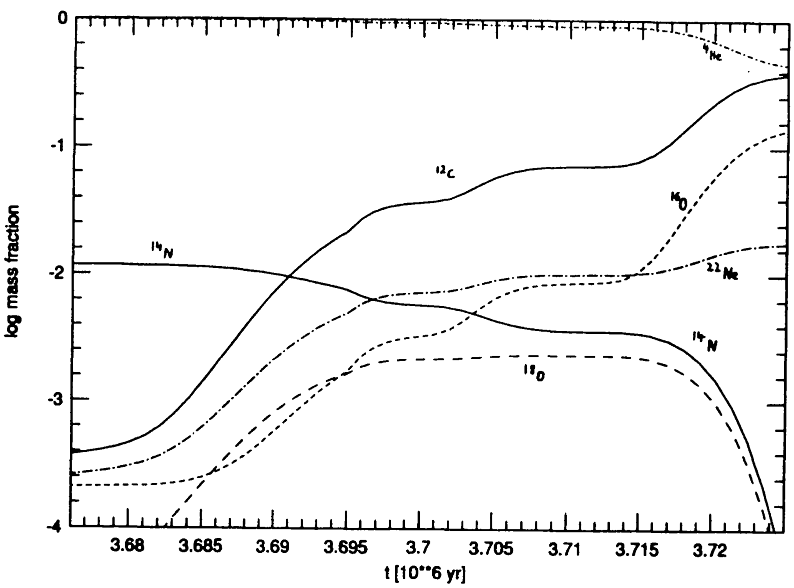

Figure 3. Surface abundances of various isotopes as a function of time of our $60\,M_\odot$ sequence for the transition phase from the WNE to the WC stage. Note the increase of the carbon abundance, well before the substantial drop of the nitrogen abundance occurs.

Concerning the ^{22}Ne abundance in WC stars, the model predictions are very coherent and almost independent on details of the stellar modeling, resulting in a surface mass fraction of 2/3 of the initial metallicity throughout the WC stage, with a small decline towards the end of core He-burning. Only during the very early WC stage, the ^{22}Ne surface abundance may by clearly smaller, but then some small amounts of nitrcgen should be present at the same time (cf. Fig. 3, and discussion below).

Finally, we want to comment on the "WN+WC" spectral type, which has been recently identified as being related to single WR stars and not always to WN+WC binaryes (Conti and Massey, 1989; Massey and Grove, 1989; Willis;

this volume). The implication is a partly mixed region between the helium burning convective core and the overlaying mass shells, which contain ashes of complete H-burning. Such a region is in contradiction to models involving only convection (with or without overshooting), since it performs mixing only on short timescales. Rather a slow mixing process is required in order to maintain a significant fraction of the nitrogen nuclei at sufficiently cool temperatures to prevent α-captures.

Though there may be several alternative mixing processes to be considered for this case, the molecular weight gradients almost certainly play a role in increasing the mixing timescale. The most straightforward mixing process in question is certainly semiconvection, since the layers under consideration become superadiabatic eventually. Fig. 3 shows the surface abundances as function of time for our $60\,M_\odot$ sequence — where semiconvection was included — in the transition phase from the WNE to the WC stage. Considerable simultaneous enhancements of both, carbon and nitrogen, are found over a period of $1-3{\cdot}10^4\,yr$, depending on the assumed threshold values.

Note that our semiconvection theory involves an efficiency parameter α (not to be confused with the WR mass loss parameter), to which the WN+WC timescale is found to be slightly sensitive. However, only α-values within a limited range were found to reproduce the blue-red-blue evolution of the SN 1987A progenitor (Langer et al., 1989), which furthermore was found to be close to the earlier theoretical estimate of this parameter (Langer et al., 1985). But even for values of α within a wider range, the WN+WC timescale was always found to be in the range quoted above. In comparison with a total WR lifetime of typically $5 \cdot 10^5\,yr$ (Langer, 1989), this means that about 3% of all WR stars should be of WN+WC type, a number which incidentally was found to be in agreement with the observationally based estimate of Willis (this volume), and is also compatible to the numbers discussed by Conti and Massey (1989), who find 8 out of 213 WR stars of this type.

The possible solution for the two very well defined problems of the SN 1987A progenitor evolution and of the WN+WC stars with the same convection theory gives us some confidence in this theory.

CONCLUSIONS

Though the WR progenitor evolution and the formation process of WR stars still contains many unsolved problems and their theoretical modeling involves large uncertainties, their relatively simple internal structure allows surprisingly strict model predictions and a comprehensive simulation of their time evolution, as outlined above. The presented $60\,M_\odot$ sequence discussed in the previous sections, which has been performed by use of a hydrodynamic stellar evolution code, using nuclear networks, extended opacity tables, etc., clearly demonstrates the validity and accuracy of the simplified evolution calculations.

The concept of mass dependent mass loss rates for hydrogenless WR stars, as also applied to our $60\,M_\odot$ sequence, leads to an overall agreement between computed and observed IMF averaged WR properties as discussed by Langer (1989a) and explains the basic discrepancy between standard WR evolutionary calculations and observations found by Schmutz et al. (1989).

WR stars enable us to quasi look deep inside the stellar interior, since their surface consists of matter which has been in the interior at earlier epochs. This opens the unique possibility of directly studying the consequences of mixing processes and nucleosynthesis, which has already been used in the past and will be so as more in the future as our observation techniques and our theoretical understanding of these objects improve. The "WN+WC" type stars may represent an excellent example in this respect.

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