This is undoubtedly the movement of the principal star, which has, therefore, an annual proper motion of 0″·036 in the direction of 72°.

On the Determination of Double Star Orbits from Spectroscopic Observations of the Velocity in the Line of Sight. By Arthur A. Rambaut, M.A.

(*Communicated by Sir. R. S. Ball.*)

I have shown in a paper which I read before the Royal Irish Academy in May 1886 how the relative velocity of the components of a double star is connected with the parallax and the elements of the orbit. In a later paper, published in the *Monthly Notices* of the Royal Astronomical Society for March 1890, I put the results in a more convenient form, and pointed out what velocities we might expect to find in the already known optical doubles.[1] The smallness of these velocities, however, seemed to show that, except perhaps in a very few cases, the spectroscopic observations of these stars were likely to prove extremely difficult.

At the time of writing the latter paper, although Professor Vogel had demonstrated the duplicity of *Algol*, and Professor Pickering the duplicity of β *Aurigæ* and ζ *Ursæ Majoris*, the results were so novel and startling that, although the idea occurred to me of solving the inverse problem—viz., to obtain the orbit from the spectroscopic observations alone—it did not seem to me that the observations were reliable enough to warrant the expenditure of the time which the solution of the question would require.

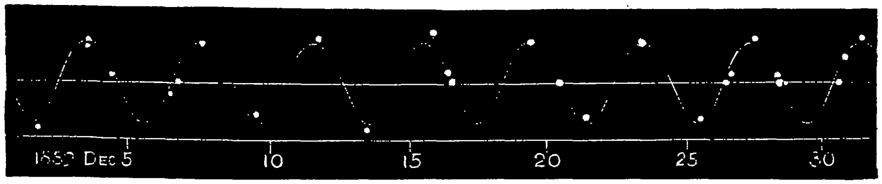
The beautiful results, however, obtained by Professor Pickering in the case of β *Aurigæ*, and published in the Fourth Annual Report of the Henry Draper Memorial, seem to throw a wholly new light on the question.

The frontispiece of this report contains five figures, the last four of which are reproductions of some of his photographic spectra, and exhibit the nature of the data on which his results depend. These prove at a glance that there is no question about the periodic displacement of the lines, and also show that the amount of the displacement is capable of measurement with very considerable accuracy.

It is, however, the first of these figures with which we are principally concerned at present. This is a curve in which the

[1] At the time of writing this I was not aware of a paper by Professor Niven in the *Monthly Notices* of the R.A.S., May 1874, dealing with the subject, in which he publishes formulæ almost identical with those given in the communications mentioned above.

time is taken as the abscissa and the relative velocity in the line of sight as ordinate. The portion of the curve given covers the whole of the month of December 1889, containing 27 measures of the displacement of the lines, and in no case does the actually



observed displacement differ from that derived from the curve by more than the accidental errors of measurement.

A glance at the curve shows that there is a periodic displacement occurring in about four days. To determine the period with greater accuracy it is necessary to take a series of observations extending over a long time, and to divide the interval which has elapsed by the number of periods which have occurred. For this purpose, of course, the longer the star is under observation, the more accurately can the period be determined. From the portion of the curve given in the report I find for the period:

$$P = 3\cdot968 \text{ days.}$$

We can now determine the form of the curve with greater accuracy. For if we take any arbitrary epoch as the zero of abscissæ and subtract once, twice, . . . *n* times P from the time, we can reduce all the observations within the same period, and thus obtain a very large number of points through which to draw the curve of velocities.

From Professor Pickering's report, treating the observations in the way just described, I obtain the following figures:—

t.	v.	t.	v.	t.	v.
0·48	−8·6	0·47	−8·6	0·74	0·0
0·94	+6·3	0·07	+8·8	0·79	+2·0
0·96	+7·6	0·20	+1·2	0·01	+8·0
0·17	+1·2	0·23	0·0	0·20	+1·4
0·67	−2·1	0·00	+7·7	0·22	0·0
0·76	0·0	0·24	0·0	0·26	−0·3
0·97	+7·1	0·48	−6·4	0·74	−0·3
0·46	−6·0	0·00	+7·8	0·81	+4·7
0·05	+8·3	0·50	−6·6	0·96	+8·0

In drawing the curve of velocities from these data there are two conditions which must be fulfilled, the reasons for which will appear when we come to determine the form of the orbit. *The* *first* of these is that the area of the curve above the axis of t shall be equal to that below the same line; *the second*, that the area contained between the curve, the axis of t and the ordinate corresponding to maximum positive velocity, shall be equal to that contained between the curve, the axis of t, and the ordinate corresponding to maximum negative velocity.

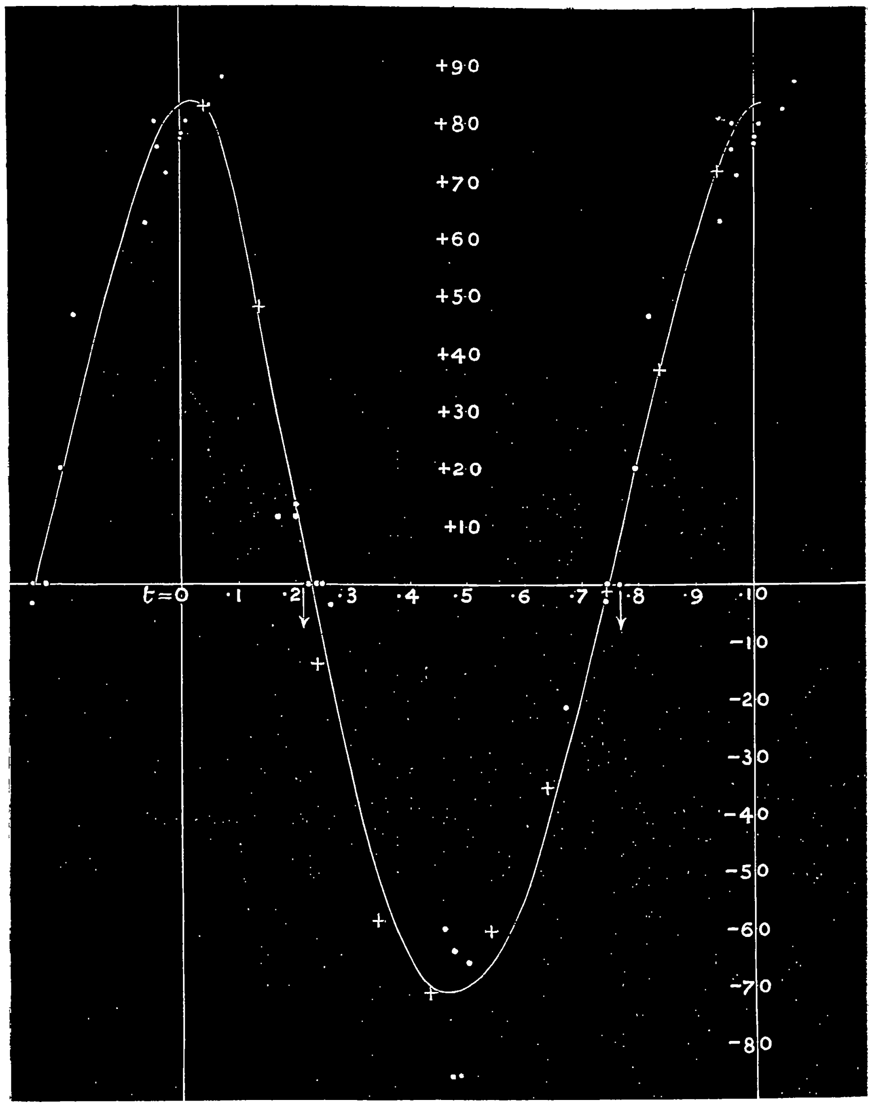


Fig. 1. Curve of Velocities.

With these two conditions in view, the curve shown in fig. 1 has been drawn from the figures given above.

I shall refer to it as *the curve of velocities.*

To Determine the Form of the Orbit.

If we take the line of sight as the axis of x, and the centre of the principal star as the origin of coordinates, we have, if V is the relative velocity of the components in the line of sight, the equation

$$\frac{dx}{dt} = \mathrm{V},$$

and consequently

$$x = \int \mathrm{V}dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The following considerations will enable us to determine the limits between which the integral is to be taken.

If we take the line of nodes as the axis of y, we have x equal to zero when the companion star is passing through this line. Now I have shown in my paper, already referred to, in the *Monthly Notices*, that when the companion is in this position the velocity in the line of sight is greatest. Hence it follows that if τ is the epoch corresponding to a maximum velocity, we have

$$x = \int_{\tau}^{t} \mathrm{V}\,dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

But when the companion is passing through the line of nodes on the other side of the primary, we have again $x=0$, and the velocity in the line of sight again a maximum, but in the opposite direction. Hence, if τ' is the epoch corresponding to this configuration, we have

$$0 = \int_{\tau}^{\tau'} \mathrm{V}dt.$$

But the expression $\int_{\tau}^{\tau'} \mathrm{V}dt$ is the area of the curve in fig. 1 between the greatest positive and the greatest negative ordinates; and since we see that this must be equal to zero, we obtain the second condition mentioned above for guidance in drawing our curve of velocities. Further, since the value of x is again zero at the line of nodes after a complete revolution has taken place, we see that we have

$$\int_{\tau'}^{\tau + \mathrm{P}} \mathrm{V}dt = 0;$$

or, what amounts to the same thing, the area of the curve of velocities, measured from the greatest negative to the greatest positive ordinate, is equal to zero. This, taken in connection with what we have already obtained, gives the first condition mentioned above, viz. that the area above the axis of t is equal to that below the same line.

In drawing this curve and computing the area contained between it and the axes, perhaps the most convenient method, and one which is quite sufficiently accurate for our present purpose, considering the nature of the data, is to plot down the observed velocities on millimetre paper to any convenient scale, taking 100^{mm} to represent the whole period. We then take a sheet of ordinary ruled writing-paper and rule it with eleven equidistant lines, so as to divide it up into a number of spaces, as shown in the table given below. For every square centimetre in fig. 1, therefore, we have a corresponding space in the table, and if we enter in each of these spaces the number of square millimetres inclosed by the curve in the corresponding square on fig. 1, we have a register of the area of the curve in a very convenient form, which enables us to see where the shape of the curve is in error, and to correct it as may appear necessary, so as to fulfil the two conditions mentioned above, without the trouble of going back and recomputing the whole area of the curve at each alteration.

	0–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9	9–10		
+90 to +80	16	...	...	...	...	...	...	...	...	4	20	
+80 to +70	74	...	...	...	...	...	...	...	...	47	121	
+70 to +60	98	3	...	...	...	...	...	...	...	84	186	
+60 to +50	100	20	...	...	...	...	...	...	14	100	234	
+50 to +40	100	38	...	...	...	...	...	...	42	100	280	
+40 to +30	100	58	...	...	...	...	...	...	68	100	326	
+30 to +20	100	77	...	...	...	...	...	1	93	100	371	
+20 to +10	100	95	1	...	...	...	...	17	100	100	413	
+10 to 0	100	100	27	...	...	...	...	40	100	100	467	2418
0 to −10	...	...	63	100	100	100	100	37	...	...	500	
−10 to −20	...	...	41	100	100	100	100	10	...	...	451	
−20 to −30	...	...	21	100	100	100	86	...	...	...	407	
−30 to −40	...	...	3	98	100	100	58	...	...	...	359	
−40 to −50	...	...	...	76	100	100	27	...	...	...	303	
−50 to −60	...	...	...	47	100	94	2	...	...	...	243	
−60 to −70	...	...	...	9	94	46	...	...	...	...	149	
−70 to −80	...	...	...	...	6	...	...	...	...	...	6	2418
	788	391	156	530	700	640	373	105	417	736	4836	

Hitherto we have only been concerned with the abscissæ, but in addition to the equation (1) we have the relation

$$x\frac{dy}{dt}-y\frac{dx}{dt}=c, \text{ a constant,}$$

which may be written in the form

$$y=cx\left[\mathrm{B}+\int\frac{dt}{x^2}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This equation will enable us to determine y if we have any means of obtaining the values of the constants c and B.

In order to determine B it is necessary to refer to the equation

$$\pi\mathrm{V}=\frac{la\sin\gamma}{\mathrm{P}\sqrt{1-e^2}}\left[e\cos\lambda+\cos(\theta-\lambda)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which I have obtained in my previous paper.

As I have pointed out there, if $\theta-\lambda=0$ or π, *i.e.* if the body is passing through the line of nodes, the velocity in the line of sight is a maximum. Let these two values of V be V_1 and V_2. We thus obtain

$$\frac{\pi}{2}(\mathrm{V}_1+\mathrm{V}_2)=\frac{la\sin\gamma}{\mathrm{P}\sqrt{1-e^2}}\cdot e\cos\lambda.$$

But when $y=0$ the line joining the two components—*i.e.* the radius vector—must be at right angles to the line of nodes, and consequently $\cos(\theta-\lambda)=0$. Hence, if V_3 is the velocity in the line of sight when in this position we have

$$\mathrm{V}_3=\tfrac{1}{2}(\mathrm{V}_1+\mathrm{V}_2),$$

or, *the ordinate* y *is zero at the moment when the velocity is a mean between the greatest positive and the greatest negative velocities.* Denoting this epoch, which we can determine from the curve of velocities, by the letter τ_0 we have

$$y=cx\int_{\tau_0}^{t}\frac{dt}{x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

We have not the means of determining from the spectroscopic observations what is the value of c, but if we take it to be any convenient factor, we shall obtain proportional values of y from equation (5), which, in connection with the values of x derived from equation (2), will enable us to construct an ellipse which is *an* orthographic projection of the real orbit on a plane passing through the line of nodes, although the angle between it and the real orbit is indeterminate.

Now this being an orthographic projection, the centre of the projected, is the projection of the centre of the real, ellipse, and the ratios of the various parts of the major axis of the real orbit are unaltered by projection. Consequently, if C (fig. 3) is the centre of the ellipse obtained as described above, P the primary, which is also the origin of coordinates, and A and B the points in which the line CP meets the ellipse, we have

$$\frac{CP}{CA} = e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

We have also from equation (4) the relations

$$V_1 = \frac{la \sin \gamma}{P\pi \sqrt{1 - e^2}} (e \cos \lambda + 1), \text{ and } V_2 = \frac{la \sin \gamma}{P\pi \sqrt{1 - e^2}} (e \cos \lambda - 1);$$

whence we obtain

$$e \cos \lambda = \frac{V_1 + V_2}{V_1 - V_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

from which λ may be determined.

Again, if $\bar{a}$ represents the semi-major axis in miles, and R the radius of the earth's orbit, we have

$$\frac{\bar{a}}{R} = \frac{a}{\pi},$$

and equation (4) becomes

$$\frac{PRV \sqrt{1 - e^2}}{l \left(e \cos \lambda + \cos (\theta - \lambda)\right)} = \bar{a} \sin \gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

from which $\bar{a} \sin \gamma$ may be determined, all the quantities on the left-hand side of the equation being already known. It will, however, be sufficient in general to take V_1 and V_2 from which to determine $\bar{a} \sin \gamma$. Thus we have

$$PRV_1 \sqrt{1 - e^2} = l\bar{a} \sin \gamma (e \cos \lambda + 1);$$

$$PRV_2 \sqrt{1 - e^2} = l\bar{a} \sin \gamma (e \cos \lambda - 1);$$

whence

$$\bar{a} \sin \gamma = \frac{PR \sqrt{1 - e^2}}{2l} (V_1 - V_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

There still remains one element which the data at our disposal enable us to determine. This is T, the epoch of periastron passage. We must, however, first determine the epoch of the passage through the node, which we shall call ϵ, and which may at once be read off the curve of velocities in fig. 1, being, in fact, the time of maximum velocity. Knowing the eccentricity we can then find the mean anomaly nt corresponding to the true anomaly λ. We have also got the mean motion equal to $\frac{2\pi}{P}$, whence we obtain

$$T = \epsilon \pm \frac{nt\,P}{2\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

In the case before us, from the velocities given above and the curve in fig. 1, which is drawn by means of them, we have the time of greatest velocity (*i.e.* ϵ) equal to $0{\cdot}02 \times P$ *later* than the epoch which we have taken as the zero of abscissæ. The latter epoch is December 2·65, and since 0·02 × P = 0·08 days, we have

$$\epsilon = \text{Dec. } 2{\cdot}73.$$

Again, measuring the area of the curve from the ordinate corresponding to this epoch, we have the following values of

$$x,\ \frac{1000}{x},\ \text{and}\ \left(\frac{1000}{x}\right)^2 :—$$

t	x	$\frac{1000}{x}$	$\left(\frac{1000}{x}\right)^2$
0·06	+ 330	+ 3·030	+ 9·18
·08	485	2·062	4·25
·10	624	1·603	2·57
·12	745	1·342	1·80
·14	845	1·183	1·40
·16	922	1·085	1·18
·18	979	1·021	1·04
·20	1015	0·985	0·97
·22	1032	0·969	0·94
·24	1030	0·971	0·94
·26	1008	0·992	0·98
·28	966	1·035	1·07
·30	905	1·105	1·22
·32	828	1·208	1·46
·34	735	1·361	1·85
·36	629	1·590	2·53
·38	511	1·957	3·83
·40	+ 385	+ 2·597	+ 6·74
0·50	− 310	− 3·226	+ 10·41
·52	448	2·232	4·98
·54	582	1·718	2·95

t	x	$\frac{1000}{x}$	$\left(\frac{1000}{x}\right)^2$
0·56	− 711	− 1·406	+ 1·98
·58	833	1·200	1·44
·60	945	1·058	1·12
·62	1045	0·957	0·92
·64	1131	0·884	0·78
·66	1204	0·831	0·69
·68	1264	0·791	0·63
·70	1310	0·763	0·58
·72	1340	0·746	0·56
·74	1354	0·739	0·55
·76	1352	0·740	0·55
·78	1334	0·750	0·56
·80	1297	0·771	0·59
·82	1243	0·805	0·65
·84	1173	0·853	0·73
·86	1088	0·919	0·84
·88	988	1·012	1·02
·90	874	1·144	1·31
·92	748	1·337	1·79
·94	− 608	− 1·645	+ 2·71

It will be noticed that the values of x from t=0·40 to t=0·50 and from t=0·94 to t=0·06 have been omitted from this table. This has been done in consequence of the fact that, x becoming very small, the corresponding values of $\frac{1000}{x}$ and $\left(\frac{1000}{x}\right)^2$ become too large to be manageable; in fact the curve in fig. 2 has a pair of infinite branches corresponding to these epochs.

The next step in the process is to plot down as ordinates the values of $\left(\frac{1000}{x}\right)^2$, which are given in the last column of this table, taking the corresponding values of t as abscissæ. We thus obtain the interpolating curve for

$$c\int_{\tau_0}^{t}\frac{dt}{x^2}$$

shown in fig. 2. To find the epoch τ_0 it is necessary to revert to fig. 1, and we find that $V_1=+84$ and $V_2=-71$, whence

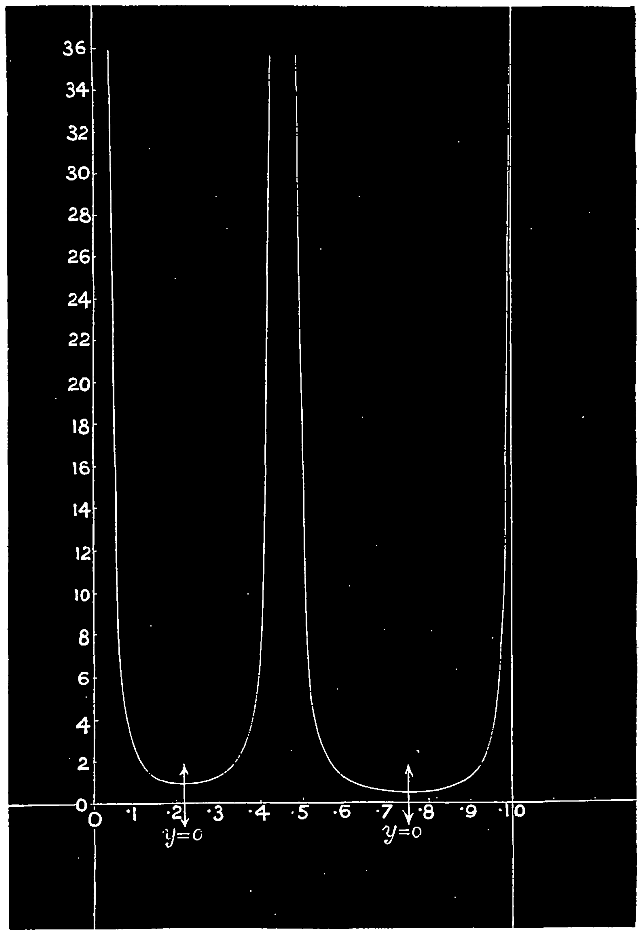


Fig. 2. Interpolating Curve for $c\int_{\tau_0}^{t}\frac{dt}{x^2}$.

$V_3=+6·5$, and the epochs corresponding to this value of V are $t=0·215$ and $t=0·761$. These two epochs are marked in fig. 2 with double-headed arrows. Calculating the areas contained between these ordinates, and the ordinates corresponding to the epochs 0·02, 0·04, &c. we get the following values for

$$c\int_{\tau_0}^{t}\frac{dt}{x^2}.$$

t	$c\int_{\tau_0}^{t}\frac{dt}{x^2}$	t	$c\int_{\tau_0}^{t}\frac{dt}{x^2}$	t	$c\int_{\tau_0}^{t}\frac{dt}{x^2}$
0·06	−163	0·36	+ 92·5	0·72	− 8
·08	106	·38	123·5	·74	3
·10	76	·40	+173·5	·76	+ 2
·12	56			·78	7
·14	41	·50	−210	·80	12
·16	29	·52	138	·82	18
·18	18	·54	102	·84	25
·20	− 8	·56	79	·86	33
·22	+ 1	·58	63	·88	42
·24	10	·60	51	·90	53
·26	19·5	·62	41	·92	68·5
·28	29·5	·64	33	·94	+90
·30	40·5	·66	26		
·32	54	·68	19		
·34	+ 71	·70	− 13		

Since the value of c is perfectly arbitrary, we may for convenience divide these numbers by 100.

We thus obtain, by means of equation (5), the following corresponding values of x and y:—

t	x	y	t	x	y
0·06	+3·30	−5·38	·56	− 7·11	+ 5·62
·08	4·85	5·14	·58	8·33	5·25
·10	6·24	4·71	·60	9·45	4·82
·12	7·45	4·17	·62	10·45	4·28
·14	8·45	3·46	·64	11·31	3·73
·16	9·22	2·67	·66	12·04	3·12
·18	9·79	1·76	·68	12·64	2·40
·20	10·15	−0·81	·70	13·10	1·70
·22	10·32	+0·10	·72	13·40	1·07
·24	10·30	1·03	·74	13·54	+0·41
·26	10·08	1·97	·76	13·52	−0·27
·28	9·66	2·85	·78	13·34	0·93
·30	9·05	3·67	·80	12·97	1·56
·32	8·28	4·47	·82	12·43	2·23
·34	7·35	5·22	·84	11·73	2·92
·36	6·29	5·82	·86	10·88	3·60
·38	5·11	6·32	·88	9·88	4·15
·40	3·85	+6·68	·90	8·74	4·63
			·92	7·48	5·12
0·50	−3·10	+6·51	·94	− 6·08	−5·47
·52	4·48	6·18			
·54	5·82	5·94			

With these values of x and y the positions marked in fig. 3 have been plotted down, and the "apparent" ellipse drawn

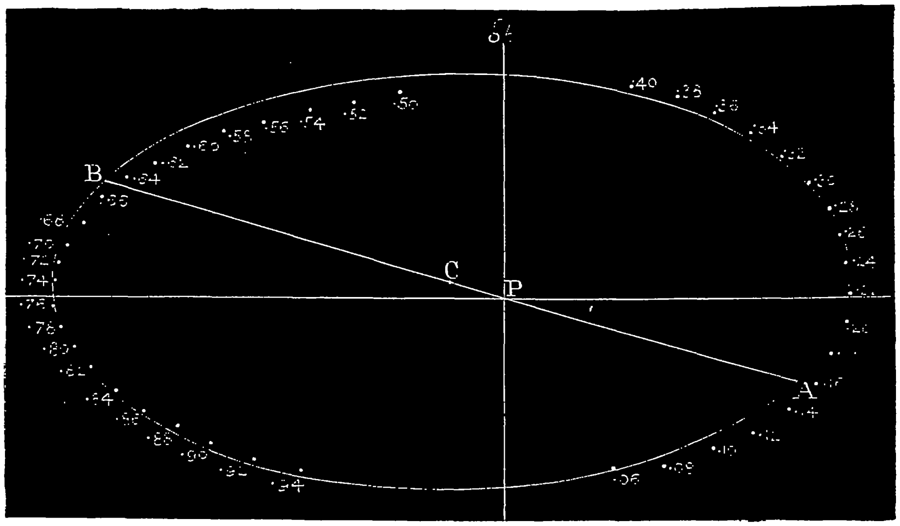


Fig. 3. "Apparent" Ellipse.

through them. Finding C the centre of this ellipse, and drawing CP to meet the curve in the point A, we have

$$\frac{CP}{CA} = 0{\cdot}156 = e$$

by equation (6).

We have also $V_1 + V_2 = 13$ and $V_1 - V_2 = 155$: hence by equation (7)

$$e \cos \lambda = \frac{13}{155} = 0{\cdot}0839.$$

and substituting the value just found for e, we find

$$\lambda = \pm\ 57^\circ{\cdot}43.$$

In computing $\bar{a} \sin \gamma$, it must be remembered that the unit in terms of which we have expressed V hitherto is perfectly arbitrary. But when the displacements of the lines are expressed in wave-lengths we can at once convert the values of V into miles. In the Henry Draper Report the wave-lengths are not given, but Professor Pickering mentions that the greatest velocity amounts to 150 miles per second. This would make $\frac{1}{2}(V_1 - V_2) = 139{\cdot}5$ miles per second. Hence we obtain, by means of equation (9),—remembering that in that equation P is expressed in years, and that consequently we must put $\frac{3{\cdot}968}{365}$ for its value—

$$\bar{a} \sin \gamma = 0{\cdot}081 \times R.$$

If we assume R to be 93,000,000 miles, this corresponds to a distance of $7\frac{1}{2}$ millions. What the actual value of $\bar{a}$ may be the spectroscope does not enable us to determine, since in most cases the value of γ will remain wholly indeterminate. In some special cases, however, such as that of *Algol*, where the variation of the lines synchronises with a variation in the brightness of the star, we may assume that γ is very nearly 90°, and that accordingly $\bar{a}\sin\gamma$ may be taken as the actual value of the semi-axis major.

We have still to compute the value of T. For this purpose we have the equation

$$\tan\tfrac{1}{2}u = \sqrt{\frac{1-e}{1+e}}\tan\tfrac{1}{2}\theta,$$

whence we obtain $u=50^\circ{\cdot}2$, when $\theta=\lambda$. Also the mean anomaly corresponding to this is $nt=43^\circ{\cdot}3$. Hence we obtain, by means of equation (10)

$$\begin{aligned}T &= \text{Dec. } 2{\cdot}65 + 0{\cdot}02\,.\,\text{P} + 0{\cdot}12\,.\,\text{P}.\\ &= \text{Dec. } 2{\cdot}65 + 0{\cdot}14\,.\,\text{P} = \text{Dec. } 3{\cdot}21.\end{aligned}$$

We thus have the following elements of the orbit of *β Aurigæ* :—

$$\text{P} = 3{\cdot}968 \text{ days}; \qquad e = 0{\cdot}156;$$
$$\text{T} = 1889, \text{ Dec. } 3{\cdot}21; \qquad \lambda = 57^\circ{\cdot}43;$$

and

$$\bar{a}\sin\gamma = 0{\cdot}081 \times \text{R} = 7{,}500{,}000 \text{ miles } \pm.$$

In order to test how far these elements will give results agreeing with the data from which they are derived, I have computed the velocity at intervals of a tenth of a period from the epoch of periastron passage. This occurs at $t=0{\cdot}14$, as we have already seen.

I accordingly obtain the following table of results :—

t	V_c	V_o	*t*	V_c	V_o
0·04	+83·0	+83·0	0·54	−60·2	−65·7
·14	+48·2	+45·2	·64	−35·2	−40·0
·24	−14·2	− 7·0	·74	− 1·3	− 2·3
·34	−58·5	−51·0	·84	+37·7	+38·0
·44	−71·0	−70·5	·94	+72·1	+72·2

in which V_c denotes the computed velocity, and V_o that read from the curve in fig. 1. The points corresponding to the values of V_c are marked in fig. 1 by small crosses, from which it will be seen that on the whole they agree fairly well with the curve of velocities.

It will also be observed that the greatest discrepancies occur in what I have called the negative part of the curve. [I may, however, remark in passing that positive and negative velocities are only relative terms in cases like the present, where the lines in the spectrum are of sensibly equal brightness, since the sign of the velocity will wholly depend on the choice we make between the lines as to which shall be considered as belonging to the primary and which to the satellite.] That these discrepancies should occur is hardly to be wondered at considering the scarcity of observations to guide us in drawing the curve in this region. In the positive part, where the observations are more numerous, the shape of the curve seems to have been hit off with very considerable accuracy, as the computed velocities lie very close to it indeed. A glance, also, will show that a very small alteration in the form of the negative portion of the curve would bring it into conformity with the computed points without doing much violence to any of the observed data, and I have no doubt that a curve might easily be drawn which would agree as well with the observed velocities as that shown in fig. 1, and at the same time give more consistent results. As, however, the velocities which I have used in constructing my curve are obtained by measuring the ordinates in Professor Pickering's report, and are, in consequence of the small scale on which his curve is given, affected with considerable errors, it does not appear advisable to push the calculation any further till the numerical results obtained by Professor Pickering are available. I have lately heard from Professor Pickering to the effect that the definitive measures upon the photographs of ζ *Ursæ Majoris* and β *Aurigæ* are in progress, and that he hopes before long to print their results with conclusions derived from them with regard to the orbits of the components of these stars. When these results appear I hope to be able to obtain more accurate values for the quantities P, T, e, λ, and $\bar{a}$ sin γ by the method pointed out above. In the meanwhile, perhaps, the results given in this preliminary notice, and the method of treating the question, may not be without value to those interested in the subject.

One point which my examination of the question seems to indicate very clearly is that the observations should be taken not so much at equal intervals of time, but as much as possible so that the points derived from them may lie equally thick along all parts of the curve of velocities, or, in other words, so that all parts of the curve may be determined with equal accuracy. In fig. 1 it will be seen that the majority of the observations are grouped about the positions of positive and negative maxima, whereas my results show that it is of equal importance to fix the exact time of the coincidence of the lines as well as the epoch at which the velocity is an arithmetic mean between its two extreme values.

I may remark, in conclusion, that in determining the orbit of stars of the *Algol* type, in which one of the components only is bright enough to form a spectral image on the photographic plate, and in which consequently its displacement has to be determined by comparing it with an artificial line, the treatment of the observations will be somewhat different. In this respect, stars such as *Sirius*, for instance, will come under the same head, if we suppose the variations in their velocity in the line of sight to be due to orbital motion. In this class of star the *whole* velocity in the line of sight is measured, as Professor Vogel has done so skilfully in the case of *Algol*, and this is composed of two parts, viz: the orbital motion relative to the centre of gravity of the system, and the absolute motion of this point, the latter of which may be assumed to be constant in magnitude and direction.

If we have sufficient and accurate enough observations to determine the curve of velocities with certainty, it will be possible to separate the orbital motion of the brighter component from the absolute motion of the system by the first of the two conditions mentioned above on p. 318. For if we plot down the velocities as in fig. 1, the curve may or may not cut the axis of t, which represents the zero of total velocity, according as the orbital is greater or less than the absolute motion. But we have seen above that the line representing the zero of orbital velocity must cut the curve, so that the areas above and below it may be equal. Accordingly, if we draw a line parallel to the axis of t, to satisfy this condition, the distance of this line from the axis of t will represent the absolute motion of the system, and the distance from it to any point on the curve will represent the velocity due to the orbital motion in the corresponding position.

Although the materials are almost ludicrously inadequate to such an investigation, I have attempted to obtain some results from the six spectrographic observations of *Algol* published by Professor Vogel in the *Astronomische Nachrichten*, No. 2,947. I find that the motion of the centre of gravity is about $-0{\cdot}7$ or $-0{\cdot}8$ German geographical miles—Professor Vogel finds $-0{\cdot}5$ on the hypothesis of a circular orbit—and that the eccentricity is something between $\frac{1}{2}$ and $\frac{1}{3}$.

It must, however, be understood that these figures are but little better than mere guesses, on account of the paucity of observations, notwithstanding the great accuracy and value of those which are available.

January 7, 1891.

Additional Note.

When the lines are equal in brightness, as in the case of *β Aurigæ*, there is an ambiguity with regard to the sign of λ, and consequently, at first sight, with regard to the sign in equation (10), since there are no means of determining whether the velocity is positive or negative in any given position: it is, in fact, positive for one component and negative for another, and there is no means of discriminating between the two. If, however, one of them is fainter than the other, we may confine our attention to the former, and we are then able to state that if the displacement is towards the red end of the spectrum the velocity is positive (*i.e.* the body is receding from us), and conversely.

In any case, however, the expression given for $e \cos \lambda$, given in equation (7), is positive if V_1 is greater than V_2. Consequently, if we take ϵ to be the time of passage through the node at which the velocity is greatest (which we may call ☊), λ will lie in the first or fourth quadrant, but there is no means of deciding, in the case of lines of equal brightness, which of these values for λ we are to choose. A positive value for λ with a rotation in one direction will give the same results as a negative value for λ with a rotation in the other direction. The question is, therefore, whether the motion is from periastron to ☊, or from ☊ to periastron. The following considerations will enable us to decide this point:—

The times of maximum velocity and the two epochs (τ_0), at which the velocity is a mean between its extreme values, mark four points on the orbit, the radii-vectores to which divide the orbit into four sectors, each of which contains a right angle. It is easy to prove that the periastron must fall in that sector whose area is least. We thus see that the time of periastron passage will fall in the shortest of the four intervals into which the period is divided by the four epochs, marked by the two maximum velocities and by the velocity which is a mean between its two maximum values. This shortest interval will always be either that immediately following or immediately preceding ϵ. In the former case we take the upper, in the latter we take the lower, sign in equation (10).

Double Star Measures at Windsor, New South Wales, 1889 *and* 1890. By John Tebbutt.

This communication contains the results of my double star measures during the years 1889 and 1890. The Grubb 8-inch equatoreal was employed throughout. The column headed "Hour-angles" gives the hour-angles between which the measures were made, and the last column the weights assigned from a consideration of the conditions under which the observations were made: 1 denotes an unusually bad condition and 5 an unusually good one. In the observations numbered 1, 6, 7, 24, 25 the components were estimated to be exactly equal. Observations 20 and 23 were made before sunset, and 2, 3, 4, 8, 13, 14, 21, 22, 29 in twilight. In observation 12 the driving-clock did not act well, and the measures were very inconsistent. The rapid angular motion of *Lalande* 4219, as indicated by the Washington, Leander McCormick, and Pinner observations, is not confirmed by the measures obtained here. The orbit of γ *Coronæ Australis* is at present far from being satisfactorily determined. Mr. Powell, in his paper published in the *Monthly Notices* for March last, states that the Windsor measures of distance are considerably greater than they should be. A practised observer will, however, be satisfied, by a mere glance at the components, that the distance is much greater than that shown by Mr. Powell's orbit. Mr. Burnham, in *Astron. Nachr.* Band 124, p. 74, gives the distance for 1889·41 as 1″·79. The following errata occur in my Double Star Results in the *Monthly Notices* for November 1889:—Page 25, *for* Lac. 5147 *read* α′ Crucis and Lac. 5147; page 26, *for* 1888·320 *read* 1888·326.