Contributions to the Optics of Mirror Systems and Gratings with Oblique Incidence. III. Some Applications

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The aberration formulas derived in paper II are checked against the well-known behavior of the Wadsworth and Eagle Mountings. A quantitative analysis of the Czerny-Turner arrangement of two mirrors and the Ebert-Fastie spectrograph is given. The compensation of the residual astigmatism for the Czerny-Turner arrangement is treated. It is found that a plane grating produces coma and astigmatism in a homocentric, converging light bundle.

THE purpose of this paper III is to show how the formulas in the preceding two parts[1,2] can advantageously be applied to the analysis of optical systems consisting of mirrors and gratings with oblique incidence. It is hoped that this will facilitate their use for a proper synthesis of such systems and help to optimize the design of instruments of this kind.

If formulas (2-1), (2-2), and (2-17)[3] are applied to the case of the Wadsworth mounting (see for instance Harrison *et al.*[4] p. 86), it follows without difficulty that there is no astigmatism and no coma for the center wavelength, for which $\sigma_0=0$. This well-known result can be considered a first confirmation for the correctness of our formulas.

Application of these formulas to the case of the Eagle mounting (see reference 4, p. 83) seems to lead to a different result. Beutler's term[5] (see reference 5, VIIIc, last paragraph), which becomes zero in the Eagle mounting and which he calls coma, is called here "spherical aberration in the sagittal plane." It follows from (2-17) with $\sigma_0=-\sigma_0'$; $\bar{s}_m=-\bar{s}_m'$; $r\cos\sigma_0/\bar{s}_m=1$ that "meridional coma" is

$$\bar{y}_c'=-\bar{h}^2\sin\sigma_0/r\cos^2\sigma_0.$$

However, this discrepancy is obviously caused by a difference of terminology, which is only a matter of convenience.

No attempt will be made to give a complete analysis of these two mountings; it would not increase our knowledge over that given by Beutler[5] and Namioka.[6,7] Some cases will be investigated for which analytical quantitative results have not yet been published, but which are interesting enough for the instrument designer to justify this effort.

* Now with General Electric Company, MSVD, Philadelphia, Pennsylvania. A preliminary report was given at the Convention of the Optical Society, October, 1959. See J. Opt. Soc. Am. 49, 1131 (1959).

1 G. Rosendahl, J. Opt. Soc. Am. **51**, 1 (1961).

2 G. Rosendahl, J. Opt. Soc. Am. **52**, 407 (1962), preceding paper.

3 The first numeral of hyphenated formula numbers refer to part I and II; see references 1 and 2.

4 G. Harrison, R. Lord, and J. Loofbourow, *Practical Spectroscopy* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1948).

5 H. G. Beutler, J. Opt. Soc. Am. **35**, 311 (1945).

6 T. Namioka, J. Opt. Soc. Am. **49**, 446 (1959).

7 T. Namioka, J. Opt. Soc. Am. **49**, 460 (1959).

The first application deals with the so-called Czerny-Turner arrangement of two mirrors of equal focal length (see Fig. 1). Its performance is compared with the symmetrical-mirror arrangement.

CZERNY-TURNER CASE, ASTIGMATISM

The sagittal image distance $\bar{s}_s'$ may be found from (2-1) as:

$$1/\bar{s}_s'=-(2\cos\sigma_0)/r+1/\bar{s}_s. \quad (1)$$

Because only concave mirrors are involved, and following Abbe's sign rule (see part I), we write

$$r=-|R| \quad (2)$$

and for the object distance in front of the first mirror with $\bar{s}_s(1)=\bar{s}_m(1)=\bar{s}(1)$

$$1/\bar{s}(1)=-(2/R)(1+\Delta);\quad \bar{s}(1)=-R/2(1+\Delta), \quad (3a,b)$$

in order to take care of a possible focus adjustment. We have then from (1) and (3):

$$1/\bar{s}_s'(1)=(2/R)[\cos\sigma_0-(1+\Delta)]. \quad (4)$$

For the second mirror:

$$\bar{s}_s(2)=\bar{s}_s'(1)-d;\quad 1/[\bar{s}_s(2)]=1/[\bar{s}_s'(1)-d], \quad (5)$$

where d is the distance between the two mirrors. Assuming that d is twice the distance of the object or image point from the mirror, we have:

$$d=-2\bar{s}_s(1)=+R/(1+\Delta). \quad (6)$$

For the second mirror, from (1) with (2)–(6) we have:

$$\frac{1}{\bar{s}_s'(2)}=\frac{2}{R}\left[\cos\sigma_0+\frac{\cos\sigma_0-(1+\Delta)}{3(1+\Delta)-2\cos\sigma_0}(1+\Delta)\right], \quad (7)$$

where $|\sigma_0(1)|=|\sigma_0(2)|=\sigma_0$ is used.

For the meridional image distance, we have from (2-2) with (2) and (3a):

$$\frac{1}{\bar{s}_m'(1)}=\frac{2}{R}\left[\frac{1}{\cos\sigma_0}-(1+\Delta)\right]. \quad (8)$$

This corresponds to (4) except that $\cos\sigma_0$ is replaced by $1/\cos\sigma_0$. We have, therefore, analogous to (7):

$$\frac{1}{\bar{s}_m'(2)}=\frac{2}{R}\left[\frac{1}{\cos\sigma_0}+\frac{1/\cos\sigma_0-(1+\Delta)}{3(1+\Delta)-2/\cos\sigma_0}(1+\Delta)\right]. \quad (9)$$

Assuming now, for instance,

$$1+\Delta=\cos\sigma_0, \quad (10)$$

that is, making $\bar{s}(1)$ equal to the "oblique focal length," we have for the astigmatic difference with (8)–(10):

$$\frac{1}{\bar{s}_m'(2)}-\frac{1}{\bar{s}_s'(2)}=\frac{2}{R}\left[T+\frac{T\cos\sigma_0}{\cos\sigma_0-2T}\right]; \quad (11)$$

$$T=1/\cos\sigma_0-\cos\sigma_0.$$

Developing (11) into series and neglecting terms with σ_0^4 and higher orders, we obtain:

$$1/\bar{s}_m'(2)-1/\bar{s}_s'(2)=(4/R)\sigma_0^2. \quad (12)$$

The same result will be obtained whenever $|\Delta|\leq\sigma_0$; that means, *astigmatism of that order* is not affected by focus adjustments of this magnitude. It is also unaffected by a comparable change of d.

It is independent of the sign of $\sigma_0(1)$ or $\sigma_0(2)$; that means astigmatism is the same for the symmetrical and antisymmetrical case. The most important conclusion follows: Astigmatism in this and similar arrangements is not eliminated without additional means. For spectrometers which show rather large astigmatism, it is worthwhile to compensate for astigmatism because of gain of energy (see, for instance, Rense and Violett[8]). An example of compensation for astigmatism will be treated below.

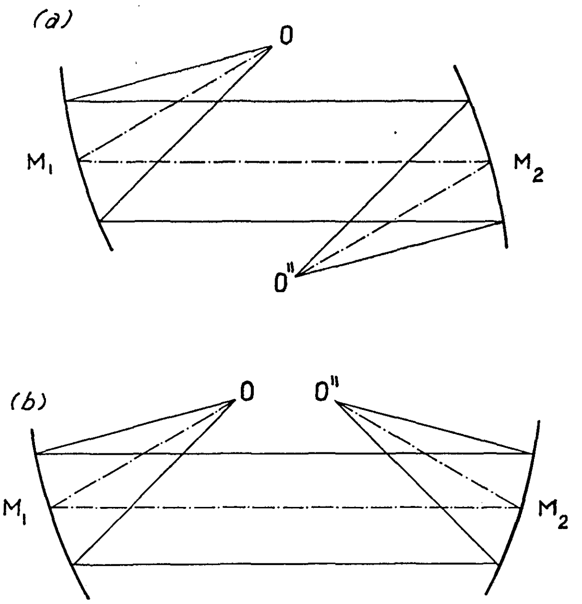

FIG. 1. Two-mirror arrangement: (a) antisymmetrical, Czerny-Turner case; (b) symmetrical case.

[8] W. A. Rense and T. Violett, J. Opt. Soc. Am. 49, 139 (1959).

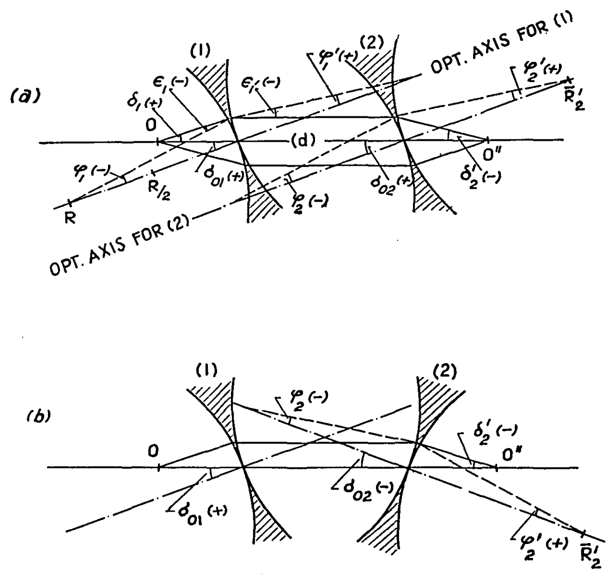

FIG. 2. Calculation of coma for the two-mirror arrangement: (a) Czerny-Turner case, (b) symmetrical case. The optical scheme is folded apart around tangent at apex of mirror surfaces in agreement with the sign convention for reflection as explained in part I.

CZERNY-TURNER ARRANGEMENT, COMA

Coma is of prime importance in this case. The optical scheme is folded apart in Fig. 2 which, in addition, makes more understandable the mathematical treatment of reflecting surfaces followed in part I.

Treating δ and φ as small angles, we have

$$\delta=-\bar{h}/\bar{s}_m. \quad (13)$$

As seen before, a fine focus adjustment does not affect astigmatism. Assuming that the same is true for coma, we may simply take

$$1/\bar{s}_m'(1)=0;\quad d=R;\quad \bar{h}(1)=\bar{h}(2). \quad (14)$$

Thus, we obtain from (3a,b) and (2-2)

$$\bar{s}_m(1)=-(R/2)\cos\sigma_0(1)=-(R/2)\cos\sigma_0(2)=-\bar{s}_m'(2), \quad (15)$$

and for (13)

$$\delta(1)=\frac{2\bar{h}(1)}{R\cos\sigma_0(1)}=\frac{2\bar{h}(2)}{R\cos\sigma_0(2)}=\delta'(2). \quad (16)$$

Coma, $\bar{y}_c''(1)$, from mirror 1 in the final image plane behind mirror 2 is with (2-22) and (2-26)

$$\bar{y}_c''(1)=\frac{\bar{s}_m'(2)}{\bar{s}_m(2)}\bar{y}_c'(1). \quad (17)$$

Substituting (2-18) into (17), we have

$$\bar{y}_c''(1)=\frac{3\bar{h}^2(1)}{2R\cos\sigma_0(1)}\tan\sigma_0(1), \quad (18)$$

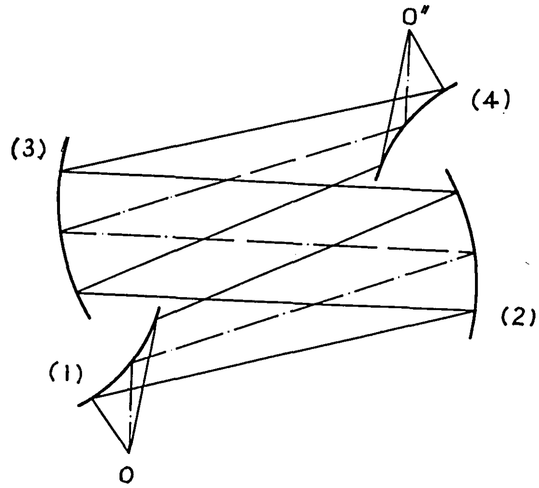

FIG. 3. Czerny-Turner arrangement with compensation for astigmatism.

where we used the relations

$$1/\bar{s}_m'(1)=1/\bar{s}_m(2)=0, \tag{19}$$

and

$$\bar{s}_m'(1)/\bar{s}_m(2)=1. \tag{20}$$

The mirror 2 coma, $\bar{y}_c''(2)$, is obtained from (2-18) and $\bar{y}_c''(2)=\bar{y}_c'(2)$,

$$\bar{y}_c''(2)=-\frac{3\bar{h}^2(2)}{2R\cos\sigma_0(2)}\tan\sigma_0(2). \tag{21}$$

For the antisymmetrical arrangement [see Fig. 2(a)]

$$\sigma_0(1)=+\sigma_0(2). \tag{22}$$

The overall coma, $\bar{y}_c''$, is then, for the *antisymmetrical case*, with (2-22), (18), and (21),

$$\bar{y}_c''=\bar{y}_c''(1)+\bar{y}_c''(2)=0. \tag{23}$$

But in the symmetrical arrangement, we have

$$\sigma_0(2)=-\sigma_0(1), \tag{24}$$

and the overall coma is, for the *symmetrical case*

$$\bar{y}_c''=+\tfrac{3}{4}R\cdot\delta^2(1)\cdot\sin\sigma_0(1). \tag{25}$$

The results are in excellent agreement with the findings of Czerny, Turner,[9] and Plettig.[10]

COMPENSATION OF ASTIGMATISM

Astigmatism can be compensated in the Czerny-Turner arrangement, without effecting the compensation of third-order coma, by two convex mirrors, (1) and (4), as indicated in Fig. 3.

The radius of the compensating mirrors, $r(1)=r(4)$, may be determined. For that purpose (2-2) and (2-1) are written:

$$1/\bar{s}_m'-C^2/\bar{s}_m=-M_m/r; \tag{26}$$

$$C^2=\cos^2\sigma_0/\cos^2\sigma_0'; \tag{27}$$

$$M_m=(\cos\sigma_0'+\cos\sigma_0)/\cos^2\sigma_0'; \tag{28}$$

and

$$1/\bar{s}_s'-1/\bar{s}_s=-M_s/r; \tag{29}$$

$$M_s=\cos\sigma_0'+\cos\sigma_0. \tag{30}$$

After astigmatism has been compensated, we will have

$$-\bar{s}_m(1)=-\bar{s}_s(1)=\bar{s}_m'(4)=\bar{s}_s(4), \tag{31}$$

and for reasons of symmetry

$$\bar{s}_m'(2)=\bar{s}_s(2)=-\bar{s}_m(3)=-\bar{s}_s(3)=\infty. \tag{32}$$

With the latter assumption, $\bar{s}_m'(3)$ and $\bar{s}_s(3)$ can be calculated with (26) and (29); and from that, $\bar{s}_m(4)$ and $\bar{s}_s(4)$. It is assumed that $r(2)=r(3)$, $\sigma_0(2)=\sigma_0(3)$, $d(1;2)=d(3;4)$, $\sigma_0(1)=\sigma_0(4)$, have been properly chosen beforehand. The angle $\sigma_0(1)=\sigma_0(4)$ should be made as large as feasible in order to keep $r(1)=r(4)$ large and their general influence upon the optical scheme small. Because of (31), we have now from (26) and (29)

$$\frac{C^2(4)}{\bar{s}_m(4)}-\frac{M_m(4)}{r(4)}=\frac{1}{\bar{s}_s(4)}-\frac{M_s(4)}{r(4)} \tag{33}$$

and

$$r(4)=\frac{M_m(4)-M_s(4)}{C^2(4)/\bar{s}_m(4)-1/\bar{s}_s(4)}=r(1). \tag{34}$$

A similar procedure for compensation of astigmatism can be used in other arrangements of the same kind.

Of course, a cylindrical mirror could be used as well; its radius is easily found. But spherical mirrors are more feasible, if good surface quality is required.

EBERT-FASTIE SPECTROGRAPH

Its general scheme follows from the Czerny-Turner arrangement (Fastie[11]). Two spherical mirrors, M_1 and M_2, with equal angles of incidence and emergence, produce parallel light falling upon a grating *Gr*. Because of the similarity to the Czerny-Turner case, no coma would be produced by M_1 and M_2 alone, but for the grating

$$1/\bar{s}_m=1/\bar{s}_m'=1/\bar{s}_s=1/\bar{s}_s'=0. \tag{35}$$

(Therefore, no *additional* astigmatism is introduced by the grating.)

For coma $\bar{y}_c'(Gr)$ at the plane grating it follows from (2-18) and with $1/r(Gr)=0$ that

$$\bar{y}_c'(Gr)=0. \tag{36}$$

The conclusion would be that the grating also does not produce coma.

[9] M. Czerny and A. Turner, Z. Phys. **61**, 792 (1930).
[10] M. Czerny and V. Plettig, Z. Phys. **63**, 590 (1930).
[11] W. G. Fastie, J. Opt. Soc. Am. **42**, 641 (1952); **42**, 647 (1952); **43**, 1174 (1953).

Fastie found, however, that rotating the grating makes re-adjustment of the second mirror for least aberrations necessary.[12] The grating, therefore, obviously has an influence upon coma. This influence consists of changing the magnification of the coma produced by the first mirror and throwing, then, the compensation of coma out of balance in the final image.

For coma $\bar{y}_c''(1)$, $\bar{y}_c''(Gr)$, and $\bar{y}_c''(2)$, we have the following expressions from (2-17), (2-18), (2-22), and (2-26):

$$\bar{y}_c''(1)=-\frac{3}{2}\frac{\bar{h}^2(1)}{R\cos^2\sigma_0(1)}\frac{\cos\sigma_0(Gr)}{\cos\sigma_0'(Gr)}\cos\sigma_0(2)\tan\sigma_0(1),\quad (37)$$

$$\bar{y}_c''(Gr)=0,\quad (38)$$

and

$$\bar{y}_c''(2)=-\frac{3}{2}\frac{\bar{h}^2(1)}{R\cos\sigma_0(2)}\frac{\cos^2\sigma_0'(Gr)}{\cos^2\sigma_0(Gr)}\tan\sigma_0(2),\quad (39)$$

where we used (15),

$$\bar{h}(2)=\bar{h}'(Gr)=\bar{h}(Gr)\frac{\cos\sigma_0'(Gr)}{\cos\sigma_0(Gr)},\quad (40)$$

and $\bar{h}(Gr)=\bar{h}'(1)=\bar{h}(1)$ [for (40) refer to (2-21)]. It can be seen that now

$$\frac{\sin\sigma_0(2)}{\cos^3\sigma_0(2)}=\left[\frac{\cos\sigma_0(Gr)}{\cos\sigma_0'(Gr)}\right]^3\frac{\sin\sigma_0(1)}{\cos^3\sigma(1)},\quad (41)$$

for complete compensation of coma, namely, for $\bar{y}_c''(1)+\bar{y}_c''(2)=0$. That means that $\sigma_0(2)$ will have to be changed if $\sigma_0(Gr)$ changes and $\sigma_0(1)$ is kept constant. The facts as expressed in (41) are in agreement with Fastie's findings.

PLANE GRATING

The result expressed in (38) has been obtained because the light bundle passing the grating was parallel. If it is converging or diverging, the result is different. Describing the converging light bundle with $\bar{s}_m=\bar{s}_s=\bar{s}$ and δ, we have

$$\bar{s}_m'=\bar{s}(\delta/\delta')(k'/k).\quad (1\text{-}17)$$

For a plane grating

$$1/r=0;\quad \varphi=\bar{h}/r=0;$$

and from (1-10)

$$\frac{k'}{k}=\frac{\cos\sigma_0'+\delta'\sin\sigma_0'}{\cos\sigma_0+\delta\sin\sigma_0}=\frac{\cos\sigma_0'}{\cos\sigma_0}(1+\delta'\tan\sigma_0'-\delta\tan\sigma_0),\quad (42)$$

for small values of δ and δ'. From (1-25) with $C_1=C_2=0$ (1-14; 1-16) it follows then for this case that

$$\delta'=\delta\frac{\cos\sigma_0}{\cos\sigma_0'}.\quad (43)$$

Introducing (42) and (43) into (1-17) yields:

$$\bar{s}_m'(\delta)=\bar{s}_m'\frac{\cos^2\sigma_0'}{\cos^2\sigma_0}\left[1+\delta\left(\frac{\cos\sigma_0}{\cos\sigma_0'}\tan\sigma_0'-\tan\sigma_0\right)\right].\quad (44)$$

This agrees with (2-13) for $\varphi=0$ and $1/r=0$. From (2-1) it follows for $1/r=0$ that

$$\bar{s}_s'=\bar{s}_s.\quad (45)$$

As may be seen from (44) and (45), and our assumption $\bar{s}_m=\bar{s}_s=\bar{s}$, a homocentric, converging, or diverging light bundle will show astigmatism as well as coma after passing a plane grating.

This shall conclude the discussion of special cases. A part IV is planned in which skew rays are treated, which will serve to investigate the aberrations in more detail.

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[12] From still unpublished investigations by Dr. Fastie. Without his kind advance information of his results, the following would have been overlooked.